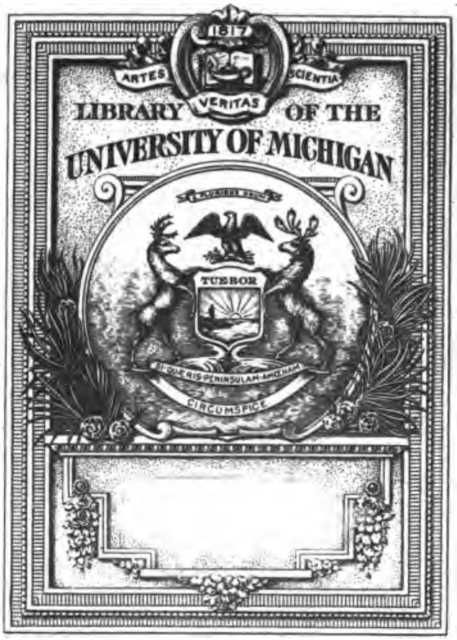
1817
ARTES
SCIENTIA
VERITAS
LIBRARY OF THE
UNIVERSITY OF MICHIGAN
TUEBOR
CIRCUMSPICE

Presented to the
University of Michigan
through Pres. Hutchins
with the compliments of
David H. Ray
May 8/11.

A
HISTORY OF MECHANICS

A BRIEF REVIEW OF THE CHIEF CONTRIBUTIONS OF THE EMINENT MASTERS OF THE SCIENCE, A TABLE OF ITS DEVELOPMENT, A CRITIQUE OF THE FUNDAMENTAL MECHANICAL CONCEPTS, AND A BIBLIOGRAPHY OF THE SCIENCE

BY

DAVID H. RAY, A.M., C.E., Sc.D.

CHIEF ENGINEER, BUREAU OF BUILDINGS, B.M., NEW YORK CITY; MEMBER OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS; ASSOC. AMERICAN SOCIETY OF CIVIL ENGINEERS; SOMETIME INSTRUCTOR IN MECHANICS AND ENGINEERING IN THE COLLEGE OF THE CITY OF NEW YORK

LANCASTER, PA.

1910

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.

PREFACE.

"As different as East from West" is a proverb. Quite apart from differences of custom, the contrast in the development of Science between the Orient and the Occident might well be considered as a measure of their diversity. The inertia and sterility in scientific production which characterize India and China is nowhere more marked than in the Science of Mechanics.

On the other hand, the ever active, ever achieving minds of the West, with persistent patience, seem at times almost on the verge of solving the cryptogram of nature and of discovering an "Open Sesame" far more potent than that of the Arabian Nights.

Certainly the achievements of the enslaved genii summoned by Alladin's Lamp, compare but poorly with the marvels wrought by the captive forces of nature controlled by our modern mechanism. While this development has been mighty, it has been slow, and it has been achieved only by hard-thinking, patient labor and experiment.

From many sources, ancient manuscripts, books, papers and letters hidden away in various libraries, the development of the fundamental principles of Mechanics may be pieced together. Much of the following short account is the result of research in this country and abroad in connection with studies for the Doctorate in Science of New York University.

With some emendation and alteration it is now offered with the hope that it may interest engineers, instructors and students. It should make clearer the nature of the Science of Mechanics and its evolution, give some idea of the power of mathematical analysis and make evident the dependence of the "practical man" upon the silent meditations of men of science.

D. H. R.

NEW YORK, 1910.

CONTENTS.

INTRODUCTION.

NATURAL SCIENCE.

PART I.

BEGINNINGS IN MECHANICS.

THE PERIOD OF ANTIQUITY, 10000 B. C. TO 500 A. D.

		PAGE.
1.	THE SCIENCE OF MECHANICS	8
2.	SCIENCE IN ANTIQUITY	12
3.	ARCHIMEDES	19

PART II.

THE MEDIÆVAL PERIOD, 500 A. D. TO 1500 A. D.

1.	THE MEDIÆVAL ATTITUDE TOWARD SCIENCE	33
2.	THE INFLUENCE OF ARABIAN CULTURE	39
3.	THE PERIOD OF THE RENAISSANCE	42
4.	THE CONTRIBUTION OF STEVINUS	45
5.	THE CONTRIBUTION OF GALILEO	52

PART III.

MODERN MECHANICS.

THE MODERN PERIOD, 1500 TO 1900.

1.	CHARACTERISTICS OF THE MODERN PERIOD	60
	HUYGENS	63
2.	NEWTON	69
3.	THE CONTRIBUTIONS OF VARIGNON, LEIBNITZ, THE BERNOULLIS, EULER AND D'ALEMBERT	77
4.	THE CONTRIBUTIONS OF LAGRANGE AND LAPLACE	106

5. Recent Contributions. The Law of Conservation 117
6. The Ether. Energy. Dissociation of Matter. 125

PART IV.

CONCLUSION.

1. Conclusions and Critique of the Fundamental Concepts of the Science 133
2. Tabular View of the Development of Mechanics 134
3. Bibliography 146

INTRODUCTORY CHAPTER.

NATURAL SCIENCE.

The word mechanics, though it indicated of old the study of machines, has long since outgrown this limited meaning and now embraces the entire study of moving bodies, both large and small, suns and satellites, as well as atoms and molecules. The phenomena of nature present to us a world of change through ceaseless motion. Mechanics is the "Science of Motion" as the physicist Kirchhoff has defined it, and has all natural phenomena for its field of investigation. Why things happen and how they happen are the questions that here present themselves.

It was a long time before the distinction between "why" and "how" was drawn, but when once the question "why" was turned over to the metaphysician and the theologian, and attention was concentrated on "*how*," then mechanics made progress. Men then began to discover "how things go," and to try their hand at invention.

It is not the purpose here to touch upon either the metaphysical or the psychological aspect of phenomena, nor the mystery of vegetable or animal activities, but to trace the development of Mechanics as a science from the earliest records to the present time, first analyzing the contributions made to it, step by step, and then touching upon their use and value.

As the French philosopher Comte first noted, three stages are apparent in the growth of human knowledge. In the first stage, man ascribed every act to the direct interposition of the Deity, in the second he tried to analyze the Deity's motives and so tried to learn "why," while in the third, men came to regard the inquiry "why" as profitless and ask "how." In this last stage, they accept the universe and are content with learning all they can of how it goes. With this last attitude, called positivism, science flourishes. Out of it grew the notion of utilitarianism,—the devotion of all energies toward the improvement of the conditions of life on earth. Though this later philosophy cannot entirely justify itself, it is commonly identified with the scientific attitude of mind.

By the long road of experience, by blunder, trial and experiment, men first gathered, it seems, ideas of things that appear always to happen together as by a necessary sequence of "cause and effect." Of the stream of appearances continuously presenting themselves, some are invariably bound together, being either simultaneous or successive, the presence or absence of the others apparently making no difference. Those having no influence may reasonably be ignored and eliminated as of no consequence. In this way, the method of abstracting from the great multitude of phenomena those that are mutually dependent seems to have been evolved.

Barbarous peoples do not possess a clear notion of sequence or of the interdependence of things. They are prone to regard the consequence of an action as accessory, as something done by an invisible being or a god. An action is performed by them, and what is commonly called by us the result is conceived by them as the simultaneous act of their god. Their medicineman is thought of, as one proficient in the art of appealing to the moods and whims of their gods propitiously. Even the Greeks and Romans, the founders of our European civilization, were accustomed to be guided in affairs of state and of the home by omens, by the flight of birds, and the inspection of the entrails of animals,—most naive examples of traditional error in the interdependence of simultaneous phenomena.

Things which we now understand to have not the slightest relation with each other were systematically confounded by the ancients. For thousands of years belief in astrology was general in Europe and the universality of the belief is attested by such words as ill-starred, disastrous, consider and saturnine, all of which are manifestly of astrological etymology. It was only very slowly and gradually, step by step, that men came to think of phenomena quantitatively rather than qualitatively, and to arrive at a more rational conception of nature through experience and reflection.

As the interrelation of things came to be more clearly perceived, people began to say they could "explain things," meaning that they had arrived at a familiarity with, and had begun to recognize certain permanent elements and sequences in the variety of phenomena. By joining these elements, they constructed a chain and attained to a more or less extensive and consistent comprehension of the relations of phenomena by a co-ordination of their permanent elements.

If these elements are linked together logically, the satisfactoriness of "the explanation" depends upon the length of the chain. The longer the chain, the further it reaches, and the more satisfied one is, the more one "understands" the matter. This is the general method of "learning things," and the information so collected may be called, as Prof. Karl Pearson has called it, an "intellectual résumé of experience." But it should be noted that it is rarely the simple correlation of things that will stand the test of experiment.

There is in this method abundant chance to go wrong. It is difficult, and especially troublesome for a beginner, untrained in this process, to decide what things really do not have effect and hence may be excluded from consideration. And if it is difficult for the beginner in science to-day, surely it was immensely more so for primitive men. Students are wont to complain of the artificiality of geometry and mechanics. Factors which they feel do make a difference in reality do not seem to them to be fully allowed for, or they are troubled by a feeling of uncertainty as to the equity of the allowance. The peculiar value of mathematical studies lies just here in the rigorous training in reasoning. Whatever a student's success with his mathematics, few make its acquaintance without receiving wholesome lessons of patient application of the intellectual method by which mankind has won its mastery over natural forces.

We may quote here to advantage Prof. Faraday.[1] "There are multitudes who think themselves competent to decide, after the most cursory observation, upon the cause of this or

[1] Lecture delivered before Royal Institution of Great Britain,—"On Education of the Judgment."

that event, (and they may be really very acute and correct in things familiar to them):—a not unusual phrase with them is, that 'it stands to reason,' that the effect they expect should result from the cause they assign to it, and yet it is very difficult, in numerous cases that appear plain, to show this reason, or to deduce the true and only rational relation, of cause and effect.

"If we are subject to mistake in the interpretation of our mere sense impressions, we are much more liable to error when we proceed to deduce from these impressions (as supplied to us by our ordinary experience), the relation of cause and effect; and the accuracy of our judgment, consequently, is more endangered. Then our dependence should be upon carefully observed facts, and the laws of nature; and I shall proceed to a further illustration of the mental deficiency I speak of, by a brief reference to one of these.

"The laws of nature, as we understand them, are the foundation of our knowledge in natural things. So much as we know of them has been developed by the successive energies of the highest intellects, exerted through many ages. After a most rigid and scrutinizing examination upon principle and trial, a definite expression has been given to them; they have become, as it were, our belief or trust. From day to day we still examine and test our expression of them. We have no interest in their retention if erroneous; on the contrary, the greatest discovery a man could make would be to prove that one of these accepted laws was erroneous, and his greatest honour would be the discovery. . . .

"These laws are numerous, and are more or less comprehensive. They are also precise; for a law may present an apparent exception, and yet not be less a law to us, when the exception is included in the expression. Thus, that elevation of temperature expands all bodies is a well-defined law, though there be an exception in water for a limited temperature; we are careful, whilst stating the law to state the exception and its limits. Pre-eminent among these laws, because of its simplicity, its universality, and its undeviating truth, stands that enunciated by Newton (commonly called the law of gravitation), that matter attracts matter with a force inversely as the square of the distance. Newton showed that, by this law, the general condition of things on the surface of the earth is governed; and the globe itself, with all upon it kept together as a whole. He demonstrated that the motions of the planets round the sun, and of the satellites about the planets, were subject to it. During and since his time, certain variations in the movements of the planets, which were called irregularities, and might, for aught that was then known, be due to some cause other than the attraction of gravitation, were found to be its necessary consequences. By the close and scrutinizing attention of minds the most persevering and careful, it was ascertained that even the distant stars were subject to this law; and, at last, to place as it were the seal of assurance to its never-failing truth, it became, in the minds of Leverrier and Adams (1845), the foreteller and the discoverer of an orb rolling in the depths of space, so large as to equal nearly sixty earths, yet so far away as to be invisible to the unassisted eye. What truth, beneath that of revelation, can have an assurance stronger than this!"

Such is the process of scientific induction. It was by linking ideas together in an orderly way, by forming and verifying hypotheses, that men finally came to the "principles," and "formulæ," which embody these general "truths" or "laws of nature." In this way knowledge has been built up, chain by chain, into a more or less complete system of the relations of things. Without asking the "why" of it all one can see "how" it goes together by running along the chains from link to link. In a word this knowledge is relative, and therefore quantitative, and that is why numbers and mathematics play so large a part in the exact sciences, and in mechanics.

The guiding principle in all this is the belief in the constancy of the order of nature founded on the experience of the human race. On this belief are based all scientific calculations and deductions. This is sometimes formulated as a "Law of Causality," affirming that every effect has a sufficient cause and that the relation of cause and effect is one of invariable sequence, if not interfered with by conditions or circumstances that make the cases dissimilar.

Information thus systematized, verified and formulated into truths or general principles is called Natural Philosophy or Natural Science. The Science of Mechanics is the oldest and one of the most important divisions of Natural Philosophy. This knowledge of the interdependence and inter-relation of phenomena makes it possible to "predict" and "control" them, and keeps us from making hasty and erroneous inferences. When developed with this view, applied science or applied mechanics is the usual designation, and that such information is power to one who has the skill to apply it, need not be dwelt upon. As Herbert Spencer says in his volume on Education:[1] "On the application of rational mechanics depends the success of nearly all modern manufacture. The properties of the lever, the wheel and axle, etc., are involved in every machine —every machine is a solidified mechanical theorem; and to machinery in these times we owe nearly all production." Elsewhere he says: "All Science is prevision; and all prevision ultimately helps us in greater or less degree to achieve the good and to avoid the bad."[2]

It is not the intention here to discuss or even to enumerate the triumphs in the practical applications of mechanics. The utilization of power, of the strength of animals, the power of the wind, of waterfalls, of steam and of electromagnetic attraction, constitutes the art of machine contrivance rather than the science of mechanics. Progress in theoretical mechanics has always brought in its train an advance in machinery.

The innumerable engines for enlightenment and destruction, the cylinder-printing-press and the machine-gun which have changed and are altering the economic, social and religious prospect of nations and tribes are the direct result of the application of the principles of the science of mechanics. With further advance in theory and systematic experimentation even more revolutionizing contrivances will inevitably follow. When invention has realized the theoretical surmise that the "molecular energy" in a cup of tea is sufficient to tumble down

[1] P. 30.
[2] "First Principles," p. 15.

a town, we may expect an Age of Power ushering in wonders untold.[1]

With the philosophy that denies the existence of realities outside of the mind we shall not trouble ourselves here. Mechanics regards a "truth" or a "law" not as subjective but as objective, holding that an external world exists and that truth is a relation of conformity between the mental world of perceptions and inferences, and really existing objects and their relations. Unless this and the validity of the principle of logical inference be conceded, our science is futile. The mental processes by which the victories of Science are won are in no wise different from those used by all in daily affairs. As Huxley says: "Science is nothing but organized common sense. The man of Science simply uses with scrupulous exactness the methods which we all habitually and at every moment, use carelessly. Nor does that process of induction and deduction by which a lady, finding a stain of a peculiar kind on her dress, concludes that somebody has upset the inkstand thereon, differ in any way, in kind from that by which Adams and Leverrier discovered a new planet."

Nevertheless there will always remain certain ultimate truths which cannot be proved and which must be consideredas axiomatic and intuitive. This should not invalidate our conclusions and we will not enter upon a discussion of these questions here.

The science of mechanics has then, for its subject matter, the motion-phenomena of the universe. Its growth is coextensive with that of the race, and one of its functions is the widening of its perceptions. It is obviously a subject of primary importance, for from apparent chaos, it evolves rules and principles of practical utility, and so increases knowledge and efficiency, and consequently happiness, through power and dominion over nature.

[1]Suppose that a cup of tea (about 100 cubic centimeters) could be suddenly and completely dissociated, after the manner of the radio-active emissions of radium, into a cloud of particles with a velocity similar to radium emanations of say 100,000 kilometers a second (about one-third the velocity of light), then a simple calculation by the theoretical formula for energy, $\frac{1}{2}mv^2$, gives $\frac{1}{2} \times .1/9.8 \times \overline{100,000,000}^2 = 50,000,000,000,000$ kilogramme-meters, equal to the energy of explosion of about 500,000 tons of rifle powder, or enough energy to drive an express train around the globe a hundred times.

PART I.

1. THE SCIENCE OF MECHANICS.

The most common of all our experiences is the motion of solid bodies. No idea is more frequently with us than the idea of such movements. It seems to be the first experience of the dawning intellect and it is soon fully developed by boyhood's games of marbles and tops. Indeed, there is nothing that our imagination pictures with greater ease and readiness, than a moving speck or particle. There is therefore considerable satisfaction, and an appealing reasonableness and inevitableness in the idea of classifying phenomena on the basis of this familiar experience.

This idea and another, quite as familiar, namely, that common objects can be crushed and broken into many small particles and ground to dust so small as to seem indivisible, are fundamental, and upon them the science of mechanics, as a scheme of motions and equilibrium of particles has been built up. Masses either change their relative position or they do not. How they move, rather than why they move, is the question of Mechanics. It is especially the circumstances of motion or of rest that are the subject of investigation of the science.

In its formal presentation in textbooks, Mechanics is now defined by an American Professor, Wright, as "the science of matter, motion, and force"; by an English Professor, Rankine, as the "science of rest, motion and force"; by a German Professor, Mach, as that branch of Science which is "concerned with the motions and equilibrium of masses." These definitions do not differ essentially.

The questions at once present themselves what is force, what is matter, what is mass? Etymology does not help us. The further back one goes, the more indistinctive and general is the idea corresponding to a scientific term. The terms, matter, mass, force and weight lose precision as we trace them back. Matter leads us back to the Latin, materia, *i. e.*, substance for construction or building. Mass appears to be derived from the Greek root (Μάσσειν), to knead. So by derivation, matter means the substance or pith of a body, and mass means anything kneaded together like a lump of dough. The fundamental idea of mass is then an agglutinated lump. Weight is of Saxon derivation from a root meaning to bear, to carry, to lift. Force appears to come from the Latin root, fortia, meaning muscular vigor and strength for violence. It is an anthropomorphic concept, and is suggestive of mythology in its application to inanimate things.

All these terms are derived from words expressing distinct muscular sensations. Here in the last analysis we come back to sense-impressions. A mass is an agglutinated lump as of kneaded dough, weight is resistance to lifting, and force is something that produces results analogous to those produced by muscular exertion. We cannot analyze these simple, immediate perceptions, nor can we analyze motion. Motion is a sense of free, unrestricted muscular action. Muscular action impeded gives us our sense of force. Perhaps our primitive perception of force was muscular action under restraint or not accompanied by motion. From these sense-impressions we attain, by inference, the idea of space, *i. e.*, room to move in, and the notion of time or uniformity of sequence. Mechanics might then be crudely defined as a scheme of the relations of lumps of matter acted upon by muscular exertion or by anything that produces like effects.

Observe that we are conscious of these sense-impressions, comparatively only. We are aware of them only through change in their intensity. Here in our endeavors to comprehend and to define the ultimate elements of mechanics we have borne in upon us the relativity of knowledge. The conviction that the human intelligence is incapable of absolute knowledge is the one idea upon which philosophers, scientists, and theologians are in accord. It is a characteristic of consciousness that it is only possible in the form of a relation.

"Thinking is relationing and no thought can express more than relations," says Herbert Spencer in his Chapter on the

Relativity of Knowledge. And he concludes: "Deep down in the very nature of Life, the relativity of our knowledge is discernible. The analysis of vital actions in general, leads not only to the conclusion that things in themselves cannot be known to us, but also to the conclusion that knowledge of them, were it possible, would be useless."[1]

But though we are limited in this way we have a large field in the building of a scheme of inter-relations of the relations which comprise our conscious perceptions. This is the purpose of our science of mechanics. In general it endeavors to interpret for us the complex relativity of phenomena in terms of the most common and simplest of our experiences, namely the relativity of motion of a particle and the relativity of the divided parts of bodies.

As science progresses the ideas, mental pictures, and terms found serviceable in the earlier stages are bound to prove inadequate later. The process of reorganizing these ideas, and perfecting terminology is slow, but in it there is unmistakable èvolutionary progress.

As the philologist Nietzsche says, "Wherever primitive man put up a word, he believed he had made a discovery. How utterly mistaken he really was! He had touched a problem, and while supposing he had solved it, he had created an obstacle to its solution. Now, with every new knowledge, we stumble over flint-like petrified words."[2]

The prehistoric races probably explained phenomena by associating with everything that produces motion, some invisible god whose muscular strength was the force of wind, wave or waterfall. We find in all languages, survivals of this in the genders ascribed to things inanimate. Indeed, one can dig out of philology and mythology a petrified primitive natural philosophy.

To-day we sometimes hear that all phenomena of the material world are explainable, in terms of matter, motion, and force, or by the whirl of molecules. One may endeavor to make this a truism by defining matter as anything that occupies

[1]Spencer, "First Principles," Chapter IV.
[2]Nietzsche, "Morgenröte," vol. 1, 47.

space, and by defining force as any agent which changes the relative condition as to rest or motion between two bodies, or which tends to change any physical relation between them, whether mechanical, thermal, chemical, electrical, magnetic, or of any other kind. But here one does not say what force is, nor what matter is. The chain hangs in the air; it does not begin or end anywhere, but the relation of the links is apparent and serviceable. Indeed, the idea of force is still fundamentally the same, it is still an *agent*, as was the ancient nature-god, though much less definite, nor does it help matters to subdivide force and mass.

The idea of force as a latent unknown cause is a historical survival of our primitive conceptions and undergoes transformation with the idea of force as a "circumstance of motion," which was developed about the year 1700. It is now held by some that force is a purely subjective conception. For example, Tait says in his "Newton's Laws of Motion": "We have absolutely no reason for looking upon force as a term for anything objective; we can, if we choose, entirely dispense with the use of it. But we continue to employ it; partly because of its undoubted convenience, mainly because it is essentially involved in the terminology of Newton's Laws of Motion, which still form the simplest foundation of our subject. It must be remembered that even in strict science we use such obvious anthropomorphisms as the 'sun rises,' 'the wind blows,' etc."

Yet though there may be no such reality as force, mechanics will probably long continue to be known as the dictionary defines it, as "the science which treats of the action of forces on bodies, whether solid, liquid or gaseous." We do not disparage the use of the idea and term force; we shall have occasion to use them often. But it should be noted that an evolution in terminology is involved in the evolution of science.

Such changes in conception and in terminology are inevitable. They are essential characteristics of progressive science which seeks continually to improve the definiteness of relation between phenomena by making clearer vague connections, or by discovering new relations. The relations formerly classed as acoustic, luminous, thermal, electric, magnetic and chemical expressing certain constant connections of antecedents and consequents are now generally expressible with exactness in the terms of the science of mechanics which is built on the familiar notions of motion and divisibility.

As a matter of convenience, the science has come to be divided into Phoronomics or Kinematics, the study of pure motion without reference to the nature of the body moved, or how the motion is produced, and Dynamics, the "science of force," "the study of the push or pull of bodies," or "the science of the properties of matter in motion." It is evident that in some cases the "forces balance," giving the condition of rest; this branch of the study is called Statics. The study of unbalanced forces producing motions of various kinds is called Kinematics. These divisions are purely arbitrary and were made late in the development of the subject. Historically, the study of Statics, or of bodies relatively at rest, was the first to be undertaken for obvious reasons.

2. THE SCIENCE OF MECHANICS IN ANTIQUITY.

It is the verdict of conservative geologists and physicists that the earth's crust is at least 25,000,000 years old, that period of time being required for the deposit of the depth of about 50,000 feet[1] of sedimentary rocks that research discloses; and it is the opinion of conservative authorities that rude communities of men were dwelling in the broad alluvial valleys of the Nile, Euphrates, Ganges, Hoang-Ho, (perhaps also on the ancient Thames-Rhine system), as early as 25,000 years ago. The subsidence of these broad rivers into narrower channels left exposed fertile plains in their old bottoms and islands in the estuaries, which favored the development of progressive communities.

This was particularly true of the Euphrates valley and along the Nile, where the wild wheat and barley offered food and made life a less severe struggle for existence. Here perhaps the first rude camps and villages were developed. But even

[1]130,000 feet is the average figure suggested by Dr. E. Haeckel—p. 9 "Evolution of Man," Vol. 2.

these early communities were probably in possession of rude tools and weapons. Darwin[1] cites instances of tools used by animals and we must imagine that even the very earliest communities of men were acquainted with such crude mechanical appliances as the lever and cord.

The researches of geologists and archæologists present innumerable stone wedges, flint axes, bone and horn implements, and primitive tools found in graves of the stone age, or on the site of ancient cave and lake dwellings, indicating extensive mechanical experience in prehistoric times.[2] An instinctive familiarity through long experience, with some of the common natural processes, and a knowledge of crude cutting and grinding tools must then be accepted as very ancient, at least twelve or fifteen thousand years old.

This must be distinguished, however, from a mechanical theory of science, which is the product of reflection. The latter was a very slow and gradual evolution. From a long experience of measuring and bartering, a knowledge of numbers probably arose, and then a more definite knowledge of the simple mechanical devices was developed. From these, by reflection and generalization, rules and principles were evolved. In the ancient Sanskrit language the word from which "man" comes, appears to mean to estimate, to measure. Man first became conscious of himself, it appears, therefore, as the being who measures and weighs, compares and reflects.

Wedges, pulleys, windlasses, oars and the lever in various forms were used before any rule for them was conceived of; and then the rules for centuries remained but disjointed unrelated statements of experience. Only very, very slowly were they mastered and made into a body of mechanical knowledge. As this process proceeded, the fetishism and mythology invented to explain natural phenomena declined before a more rational and logical group of mechanical principles. But traces of it long survived. For example, the idea that "nature abhors a vacuum" is a late survival of such

[1] The Descent of Man, Chapter III, "Tools and Weapons used by Animals."

[2] Prehistoric Times, Sir J. Lubbock; Ancient Stone Implements, Evans; Man and the Glacial Period, D. F. Wright; Man's Place in Nature, T. H. Huxley; Origin of Species, etc., C. Darwin.

fanciful conceptions, and was cited as late as 1600 A.D. But for Science, as Spencer says, we should be still worshipping fetishes; or with hecatombs of victims be propitiating diabolical deities.

It seems that it was only among the people of the Eastern Mediterranean coast that a true science of mechanics was developed. There is no evidence to show that among any of the peoples of the Far East any true science of mechanics was even begun. Indeed some of the people of the yellow and darker races still live in the stone or bronze age. Certainly the whole development of the science as we have it is European. Of the world's population of 1,500,000,000, the 200,000,000 of Europe and the 100,000,000 of America who have a grasp on mechanical science are in control. Half of Asia's 700,000,000 are held subject by Europe's Science, and the destiny of the other half is the topic of the hour.

To the Babylonians and Phœnicians, skilled in measuring, in plane surveying, in keeping accounts, and in seafaring, the science of Europe is traced back. Centuries before the era of Greece, the Phœnicians had developed a crude astronomy and were practicing and slowly improving the common mechanic arts and trades. They are not to be credited with originating them however, for scholars have traced these people back to a mingling of tribes of primitive Semetic and Aryan stock which took place in the Tigris-Euphrates region of Asia, about 8000–10000 B.C.

Here a remarkable civilization of teeming cities had developed by 5000 B.C. The trials and troubles, the institutions, arts, literature, and the wail of the prophets, the complete life history of growth and decay of these cities may be read in the cuneiform inscriptions on the clay tablets in the British Museum. With the shifting of the trade routes to the north and west, through the Dardanelles, their prosperity declined and they passed out of existence.

Perhaps the oldest relic of their mechanical arts is the splendid tablet or "stele" set up in the temple of Lagash by Eannatum (c. 4200 B.C.). One side shows the king in his chariot leading his army to victory, the other shows the wreck and ruin of the vanquished whose mangled corpses are left to the vultures. The great king of these people, Sargon I (c. 3800 B.C.), is said to have extended his conquests westward as far as the Island of Cyprus, the land of copper. Bartering expeditions then as now spread information and developed the arts and trades. As early as 3000 B.C. the Egyptians seem to have become a power.

It seems, then, that the European development of mechanics as a science is founded on at least 3,000 or 4,000 years[1] development of the recognized mechanic arts and trades,[2] and it is probable that it began with the systematizing of craft experience and the formulation of this experience in connection with the instruction of apprentices.

Reflection on methods, and endeavors to train novices by the experience and mistakes of older craftsmen, formed a sort of groundwork of experience, and tended to develop a nomen-

[1]The Egyptian pyramid of Cochrome is referred by archæologists to the first dynasty of Manetho, 3600 B.C., making it fifty-five centuries old. It exhibits well developed skill in the trades, "dating from a time nearly coincident, according to Biblical authority, with the creation of the world itself (3761 B.C.)"—Reber, History of Ancient Art, p. 3. See also, Petrie; Maspero; Perrot and Chipiez.

[2]The Egyptians' sculptured wall reliefs and wall paintings exhibit considerable specialization in the trades several thousand years B.C. As for the Greeks, the picture of Vulcan's smithy in Iliad XVIII is that of a most busineslike and efficient shop. There is no mention of iron or steel, but it indicates the tools employed 1000 B.C.

So speaking he withdrew, and went where they lay
The *bellows*, turned them toward the fire, and bade
The work begin. From *twenty bellows* came
Their breath into the *furnaces*,—a blast
Varied in strength as need might be; . . .
And as the work required. Upon the fire
He laid impenetrable *brass*, and *tin*
And precious gold and silver; and on its *block*
Placed the *huge anvil* and took the ponderous *sledge*
And held the *pincers* in the other hand.
. . . .
When the great artist Vulcan saw his task
Complete, he lifted all that armor up
And laid it at the feet of her who bore
Achilles. Like a falcon in her flight,
Down plunging from Olympus capped with snow,
She bore the shining armor Vulcan gave.

William Cullen Bryant's Translation.

clature, and a set of rules. This indicates in its very genesis the practical and economical character of mechanical science. It generalizes experience. It is not only a mental labor-saving device, but also a guide to the fashioning of physical labor-saving apparatus.

Mechanics began, then, with the theory and rules of the trades. The very common origin of its twin-brother geometry, is seen on translating this Greek word into English: Γεωμετρία, ἡ,—the science of measuring the earth.[1] Herodotus attributes the origin of this science to the necessity of resurveying the Egyptian fields after each inundation of the Nile and refers to the system of taxation of Rameses II (c. 1340–1273 B.C.), which required such survey. Early geometry was therefore a crude theory of land surveying. Its abstractions and rules were brought to bear upon mechanical problems and there followed that intimate connection in the development of these sciences which has been so useful. Formal mechanics has indeed been called by one of the masters,[2] a geometry of four dimensions, *i. e.*, the three spatial dimensions and time.

The Ahmes papyrus of the British Museum, "Directions for Obtaining Knowledge of all Dark Things" (about 2000 B.C.), is perhaps the oldest treatise on arithmetic in existence. The Egyptians appear to have had manuscripts on arithmetic as early as 2500 B.C. But what every school boy is now taught was then a dark mystery known to but a few priests and scribes. The hieroglyphic numerals are a vertical line for 1, a kind of horse shoe for 10, a spiral for 100, a pointing finger for 10,000, a frog for 100,000 and the figure of a man in the attitude of wonder for 1,000,000; a rather hopeless notation for mechanical calculations from the modern point of view.

Building on the accumulations of Egyptian and Phœnician civilization, the Greeks began the Science of Mechanics by applying in the trades the rules of geometry and the inductive and deductive methods of thought. They labored under the

[1] Pickering's Greek Lexicon; Aristoph. Nub. 202; Th. γέα and μέτρον; also Herodt.

[2] Joseph Louis Lagrange (1736–1813).

erroneous conceptions of nature taught in their mythological religion, and they were further handicapped by the notion that it was not necessary to investigate nature at first hand, but that the scheme of things could be evolved by ratiocination.

Mechanics as a science may be said to begin with the Greeks, as they formulated the first principle of mechanics. But their speculations were limited to problems of equilibrium, that is, to Statics. They never evolved any rational theory of moving bodies. Dynamics was unknown to them and did not take form as a branch of mechanics until about 1600 A.D. The great bulk of the correct *theory* of mechanics known in antiquity is commonly attributed to Archimedes. Before considering his work, it will be profitable to glance at the work of several of his predecessors.

Thales, probably of Greek and Phœnician ancestry, tradition declares, brought the art of geometry from Egypt into Greece about 600 B.C. He taught half a dozen theorems by the inductive method. Proclus a Greek teacher of about 450 A.D., speaks of him as the father of geometry in Greece, and declares that he learned it in Egypt.

His method was later extended by Pythagoras who, about 500 B.C., prepared two books of geometry on the deductive plan. He appears to have been the first to separate clearly the studies of geometry and of numbers. By pointing out that quantity is incommensurable, but that measure of quantity or a unit may be enumerated or counted, he drew the distinction between geometry and arithmetic, and set apart the study of numbers or arithmetic as a branch of mathematics.

One finds it difficult to realize the mysticism and magic with which so commonplace an idea as a number was then mingled. Pythagoras regarded numbers as having celestial natures, the even numbers as feminine and the odd as masculine![1]

Hippocrates (420 B.C.) invented the method of reducing one theorem to another for proof instead of going back to the axioms with each proposition; while Eudoxus (355 B.C.) invented proportion and devised the method of exhaustions,

[1]"The Philosophy of Arithmetic," Dr. Edw. Brooks.

one form of the idea of limits which he applied in geometry.

About 300 B.C., Euclid collected and systemized the geometry and number-work of his time, invented some new propositions and made a volume on the "Elements of Geometry." This work of fifteen books remained the standard text-book of geometry the "Euclid," of the following twenty centuries. The work gives rules for the geometrical construction of various figures, as well as the proof of numerous theorems. He also wrote a volume on Conics and Geometrical Optics.

Aristotle (384–322 B.C.), the famous Greek teacher, often mentioned as one of the founders of Science, is notable for his voluminous writings on philosophy, on natural history and on geometry, which in part directed attention to the study of nature by direct observation. But there is no doubt that his teaching on the theory of motion and some of his notions on equilibrium were erroneous. His great reputation as a natural philosopher gained acceptance for some of his opinions for eighteen centuries after his time, and as they were wrong, this was a great impediment to the development of the science of mechanics. Even as late as 1590, Galileo felt the strength of the partisans of the erroneous Aristotelian philosophy who forced him from the University of Pisa.

By 200 B.C., four centuries after Thales, the Greeks had brought their geometry to a high stage of perfection. Apollonius, of Perga (d. 205 B.C.), published about this time a treatise on conic sections and geometry containing over four hundred problems which left little for his successors to improve. His problem, "to draw a circle tangent to three given circles in a plane," found in his tretaise on "Tangency," has baffled many later mathematicians.

His studies on astronomy were the basis of Ptolemy's exposition of planetary motions and his goemetry has been discovered in two distinct Arabic editions, indicating its influence on Moorish mathematics of the ninth and tenth centuries. He also wrote on methods of arithmetical calculation and on statics, but this work is overshadowed by that of his contemporary Archimedes, who appears to have co-ordinated the scattered information on statics and to have contributed largely to it.

What Euclid did for geometry Archimedes tried to do for Statics. In this he was in part at least successful. For he developed a body of correct mechanical doctrine which still finds place to-day in our elementary text books of this science.

3. THE CONTRIBUTIONS OF ARCHIMEDES.
(287–212 B.C.)

Archimedes, the greatest mathematician of antiquity, the son of a Greek astronomer, had the advantage of a good training in the schools of Alexandria, and then retired to Syracuse in Sicily, where he devoted himself to the study of mathematics and mechanics.

We know his work through the manuscripts and the books which have come down to us, and by references to him in the classics which give us some slight additional data. Some of his writings we have in the original Greek, while others exist only in the Latin or Arabic translation. They may be briefly summarized as follows:

Extant Works.

1. On the Sphere and Cylinder.
   Two books containing sixty propositions relative to the dimensions of cones and cylinders, all demonstrated by rigorous geometric proof.
2. The Measure of the Circle.
   A book of three propositions. Prop. I proves that the area of a circle is equal to a triangle whose base is equal to the circumference and whose altitude is equal to the radius. Prop. II shows that the circumference exceeds three times the diameter by a fraction greater than 10/70 and less than 10/71. Prop. III proves that a circle is to its circumscribing square nearly as 11 to 14.
3. Conoids and Spheroids.
   A treatise of 40 propositions on the superficial areas and volume of solids generated by the revolution or conic sections about their axis.
4. On Spirals.
   A book of 28 propositions upon the curve known as the

spiral of Archimedes which is traced by a radius vector whose length varies as the angle through which it turns.

5. On Equiponderants and Centers of Gravity.

Two volumes which are the foundation of Archimedes' theory of Mechanics. They deal with statics. The first book contains fifteen propositions and eight postulates. The methods of demonstration are those often given to-day for finding the center of gravity of—

(*a*) any two weights,
(*b*) any triangle,
(*c*) any parallelogram,
(*d*) any trapezium.

The second volume is devoted to finding the center of gravity of parabolic segments.

6. The Quadrature of the Parabola.

A book of 24 propositions demonstrating the quadrature of the parabola by a process of summation—a kind of crude integration.

7. On Floating Bodies.

A treatise of two volumes on the principles of buoyancy and equilibrium of floating bodies and of floating parabolic conoids.

8. The Sand Reckoner, or Arenarius.

A book of arithmetical numeration which indicates a method of representing very large numbers. He indicates that the number of grains of sand required to fill the universe, is less than 10^{63} It contains an idea which might have been developed into a system of logarithms.

9. A collection of Lemmas,—fifteen propositions in plane geometry.

Archimedes is also credited with these lost books, though some authorities dispute the fact that he ever wrote such volumes; that he worked upon the subjects there is little doubt.

Lost Works.[1]

1. On Polyhedra.
2. On the Principles of Numbers.
3. On Balances and Levers.
4. On Center of Gravity.
5. On Optics.
6. On Sphere Making.
7. On Method.
8. On a Calendar or Astronomical Work.
9. A Combination of Wheels and Axles.
10. On the Endless Screw or Screw of Archimedes.

Archimedes is to be credited with the development of a theory of the lever, the principle of buoyancy, the theory of numbers and numeration. He was the first to apply correctly geometry and arithmetic to mechanical problems of equilibrium, and he thus founded the science of applied or mixed mathematics. He founded and developed the theory of statics in reference both to rigid solids and fluids, but he by no means completed it. He developed no correct theory of dynamics. The following quotations from his book on Equilibrium, or the "Center of Gravity of Plane Figures," give an insight to his mental attitude and an idea of his method of approaching problems in mechanics.

Book I.

"I postulate the following:

1. "Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline toward the weight which is at the greater distance.

2. "If, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium but incline toward that weight to which addition is made.

3. "Similarly, if anything be taken away from one of the weights, they are not in equilibrium but incline toward the weight from which nothing was taken.

[1]Accounts of the recently discovered "lost works" of Archimedes will be found in the following periodicals: *Hermes*, vol. 42; *Bulletin of the American Mathematical Society*, May, 1908; *Bibliotheca mathematica*, vol. 7, p. 321.

4. "When equal and similar plane figures coincide if applied to one another, their centers of gravity similarly coincide."

5. "In figures which are unequal but similar the centers of gravity will be similarly situated. By points similarly situated in relation to similar figures, I mean points such that, if straight lines be drawn from them to the equal angles, they make equal angles with the corresponding sides."

6. "If magnitudes at certain distances be in equilibrium (other) magnitudes equal to them will also be in equilibrium at the same distances."

7. "In any figure whose perimeter is concave in (one and) the same direction the center of gravity must be within the figure." This is the way he proves the equilibrium of the lever.

"Proposition 1."

"Weights which balance at equal distances are equal."

"For, if they are unequal, take away from the greater the difference between the two. The remainders will then not balance—(Postulate 3); which is absurd."

"Therefore the weights cannot be unequal."

"Proposition 2."

"Unequal weights at equal distances will not balance but will incline toward the greater weight."

"For take away from the greater the difference between the two. The equal remainders will therefore balance (Postulate 1). Hence if we add the difference again the weights will not balance but will incline toward the greater (Postulate 2)."

Proposition 3.

Proves that weights will balance at unequal distances, the greater weight being at the lesser distance, by a similar kind of reasoning.

Proposition 4.

Shows similarly that two equal weights have the center of gravity of both at the middle point of the line joining their centers of gravity.

Proposition 5.

Proves, if three equal magnitudes have their centers of gravity on a straight line at equal distances, the center of gravity of the system will coincide with that of the middle magnitude. He then proves,

Propositions 6–7.

Two magnitudes, whether commensurable (Prop. 6) or incommensurable (Prop. 7) balance at distances reciprocally proportional to the magnitudes.

1. Suppose the magnitudes A, B to be commensurable and the points A, B to be their centers of gravity.

Let DE be a straight line so divided that at C

$$A : B = DC : CE$$

We have then to prove that, if A be placed at E and B at D, C is the center of gravity of the two taken together.

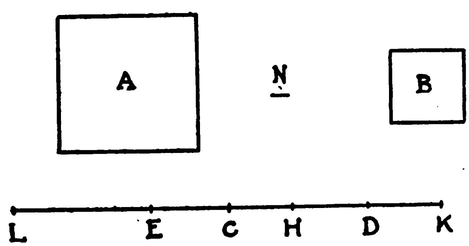

FIG. 1.

Since A and B are commensurable, so are DC, CE. Let N be a common measure of DC, CE. Make DH, DK each equal to CE and EL (on CE produced) equal to CD. Then $EH = CD$. Since $DH = CE$ therefore LH is bisected at E, as HK is bisected at D.

Thus LH, HK must each contain N an even number of times.

Take a magnitude O such that O is contained as many times in A as N is contained in LH whence

$$A : O = LH : N$$

But

$$\begin{aligned} B : A &= CE : DC \\ &= HK : LH \end{aligned}$$

"Hence $B : O = HK : N$, or O is contained in B as many times as N is contained in HK."

"Thus O is a common measure of A, B. Divide LH, HK into parts each equal to N, and A, B, into parts each equal to O. The parts A will therefore be equal in number to those of LH, and the parts of B equal in number to those of HK. Place one of the parts of A at the middle point of each of the parts N of LH, and one of the parts of B at the middle point of each of the parts N of HK.

"Then the center of gravity of the parts of A placed at equal distances on LH will be at E, the middle point of LH (Proposition 5, Cor. 2), and the center of gravity of the parts of B placed at equal distances along HK will be at D the middle point of HK.

"Thus we may suppose A itself applied at E, and B itself applied at D."

"But the system formed by the parts O of A and B together is a system of equal magnitudes even in number and placed at equal distances along LK, and, since $LE = CD$ and $EC = DK$, $LC = CK$ so that C is the middle point of LK. Therefore C is the center of gravity of the system ranged along LK.

"Therefore A acting at E and B acting at D balance about the point C."

The incommensurable case.

"Suppose the magnitudes to be incommensurable and let them be $(A \pm a)$ and B respectively. Let DE be a line divided at C so that

$$(A + a) : B = DC : CE$$

"Then, if $(A + a)$ placed at E and B placed at D do not balance about C, $(A + a)$ is either too great to balance B or not great enough."

"Suppose, if possible that $(A + a)$ is too great to balance B.

Take from $(A + a)$ a magnitude smaller than the deduction which would make the remainder balance B, but such that the remainder A and the magnitude B are commensurable.

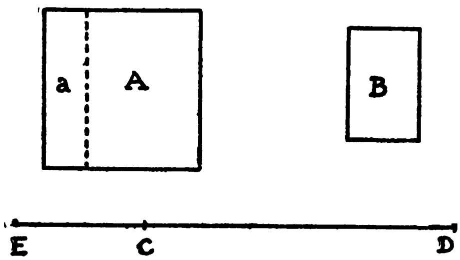

FIG. 2.

"Then, since A, B are commensurable and

$$A : B < DC : CR$$

A and B will not balance (Prop. 6) but D will be depressed.

"But this is impossible since the deduction A was an insufficient deduction from $(A + a)$ to produce equilibrium, so that E was still depressed.

"Therefore $(A + a)$ is not too great to balance B; and similarly it may be proved that B is not too great to balance $(A + a)$.

"Hence $(A + a)$, B taken together have their center of gravity at C."

Thus it is seen that the demonstration rests upon the axiom that equal bodies at the ends of equal arms of a rod supported at its middle point will balance each other. From this he proves that the bodies will be in equilibrium when their distances from the fulcrum are inversely as their weight, and all his determinations are based on these propositions. All his investigations are limited to the case of forces perpendicular to straight lever arms, he does not appear to have grasped the idea of "moments" or of "equal work" up and down. These conceptions were not fully attained until eighteen centuries later.

To Archimedes also belongs the fame of establishing the principle of buoyancy commonly known as Archimedes' principle. The account of this discovery given by Vitruvius in De Architectura, Liber IX, is as follows: "Although Archimedes discovered many curious things proving his great intelligence, that which I now narrate is the most remarkable. Hiero, when he obtained the regal power in Syracuse, having on the happy turn of his fortunes decreed a votive crown of gold to be placed in a certain temple, commanded it to be made of great value, and assigned for the purpose an appropriate weight of metal to the goldsmith. The latter in good time presented the crown to the king beautifully wrought and of correct weight.

But a report having been circulated, that some of the gold had been supplanted with silver of equal weight Hiero was indignant at the fraud, and appealed to Archimedes for a method by means of which the theft might be detected. Charged with this commission he by chance went to a bath, and on getting into the tub perceived that just in proportion that his body became immersed, in the same proportion the water ran out of the vessel. Whence catching at the method to be used in solving the king's difficulty he leapt out of the vessel in joy, and ran naked shouting in a loud voice, *εὕρηκα*, I have found it!"

It seems that Archimedes' conception was, that a body immersed in water must raise an equivalent quantity of water just as though the body lay on one arm of a balance and the water on the other arm. Buoyancy he conceived as a case of equilibrium or equipoise by a balance of weights. If the object overbalances the water displaced it sinks. These ideas he elaborated in his book on Floating Bodies.

One of his fundamental assumptions in this work is that it is an essential property of a liquid that the portion that suffers less pressure is forced upward by that which suffers greater pressure and that each part of the liquid suffers pressure from the portions directly above it, if the latter be sinking or suffer from another portion. From this he elaborates the ideas,

(1) That when a heavy body is entirely surrounded by liquid it is buoyed up or balanced in part, by a force equal to the weight of the liquid it displaces;

(2) That when bodies lighter than a fluid are wholly immersed in it, they displace an amount of liquid greater than their own weight and so if left free to adjust themselves they rise to the surface and float so that only so much of their bulk is submerged as will displace sufficient liquid to balance themselves;

(3) When a submerged body displaces a magnitude of liquid which just balances itself it is in equilibrium anywhere below the surface of the liquids.

It follows from the story of the gold and silver crown that Archimedes must have arrived at the idea of relative density or specific gravity but he could not distinguish mass and weight. The favorite word in his discussions is the abstract mathematical term magnitude by which he often seems to mean mass. But how mass gets that drag downwards, or how the force or weight is related to mass he did not attempt to explain. Certainly he presents no theory on the subject in any of his extant works.

Some convenient mechanical appliances are by tradition commonly attributed to Archimedes, notably the Archimedean screw or pump, a device said to have been invented by him while in Egypt for use in the irrigation works. His practical inventions indicate that he, in common with all the eminent masters, was not so lost in his theoretical studies as to be out of touch with practical affairs.

It should be noted that the geometry of Euclid was a geometry of forms and positions whereas that of Archimedes was a geometry of measurement. This new trend is seen in the attention that Archimedes gives to problems of the quadrature of curvilinear plane figures (such as the parabola), and to the cubature of curved surfaces. This development of geometry placed it in most intimate connection with mechanics, for progress in the latter depended upon accuracy of measurement.

Therefore we are indebted to Archimedes not only for the mechanical devices and rules commonly associated with his name, but also for having given to geometry that trend of development into a science of measurements which made it of such assistance in developing mechanics. Archimedes himself well illustrated this in the field of statics. Later when the labors of Galileo and Stevinus had developed the method of representing forces, velocities and acceleration by lines, it was by similar geometrical methods that Newton in his Principia presented the proofs of theorems in dynamics.

Ctesibius and Hero (cir. 150 B.C.) are sometimes mentioned as the successors of Archimedes. Following Archimedes' method they formulated a table of mechanical appliances setting forth the five simple principles or "simple machines," about as they are listed in our elementary textbooks of physics to-day. But though they made several practical inventions such as the forcing pump, the clepsydra and air-gun and contrived curious fountains and syphons, they do not appear to have added anything to the principles of mechanics, nor do they appear to have comprehended the theory of their mechanical appliances, except in so far as the principles of Archimedes could explain them.

There is no evidence to show that the principle of work was understood or appreciated in ancient times in spite of

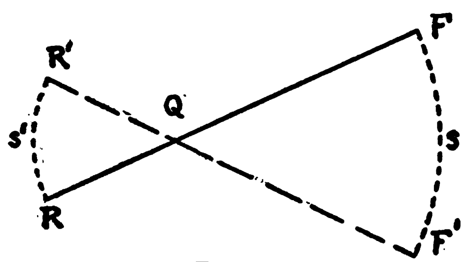

FIG. 3.

the fact that we feel almost instinctively now, that in a lever such as Fig. 1, the force times the distance it moves (*i. e.*, the work applied), is equal to the resistance times the distance it moves, (*i. e.*, the work done) if we ignore friction, and that the algebraic sum of the positive and negative work is zero. The simple equation of work, $F \times S = R \times S'$ was Chinese to Archimedes, for algebraic symbolic notation was not known in mechanics in his time. Archimedes does not appear to have attained to the conception of "moment," nor "principle of work," nor "conservation of center of gravity." He made equilibrium in the lever depend on the length of the lever-arm and the "magnitude" of the bodies hung on the ends of the lever-arms without understanding the terms moment, mass, work or weight in the modern sense.

The physical science of the Greeks was limited to calculations based upon:

(1) The law of the lever,
(2) Center of gravity,
(3) Density,
(4) Hydrostatic pressure,
(5) Arithmetical relations of tones,
(6) The law of the reflection of light.

Ancient Greece was a slave country. At the height of its glory Athens contained twenty slaves to one free citizen. The slaves were permitted no initiative and there was no incentive to mechanical invention. Indeed, the application of natural forces and the substitution of machines for slave-labor would have been viewed with alarm by all classes of the Greek state as ushering in an industrial and social revolution. Inventors and innovators therefore met scant encouragement in ancient Greece.

The government was a close corporation of capitalist citizens whose profits depended upon the slaves. Furthermore it was to the interest of the government to keep the slaves steadily employed yet not oppressively burdened. Conditions of life were favorable in the Greek peninsula, and history records very few slave insurrections. There was no urgent demand for mechanical invention and no reward for it. Only free men have an interest in the improvement of their tools and only under the laws of property and of patents is there encouragement and incentive to mechanical invention.

With the extension of the Roman power on the fall of Syracuse, in which Archimedes lost his life, conditions were not favorable to the advance of science. The Romans, a practical, commerical, military people did not advance the theory of mechanics. Their talents lay in administration rather than in science. They used the simple machines practically and successfully on land and on sea, in war and in peace, and by trireme and catapult extended their dominon over the known world. In their marvellous public works aqueducts, baths, fountains, sewers, roads, public buildings, and monuments, we find examples of the art of construction rather than of the science of engineering.

They built by "rule of thumb" and experience based on trial, using a large surplus of materials. No delicate appreciation of stresses is apparent in their architecture. It is massive, heavy and monumental, without subtlety of artistic conception or of scientific design. It is true the Romans introduced as common features in their buildings, arches, vaults and domes which were used by the Greeks, Persians and Egyptians but rarely, but they used them without theoretical calculation. Some of their buildings were supplied with running water carried in lead pipes, and were heated by hot air in tile flues but they had not grasped even the elements of hydraulics or thermodynamics. It was not till after 1600 A.D. that the principle of moments and the law of action and reaction upon which the common engineering calculations are based, were fully apprehended.

Nor did Roman philosophers and writers busy themselves with mechanical science. The works of Lucretius (95–52 B.C.), Vitruvius (85–26 B.C.), Seneca (2–66 A.D.) and Pliny (23–79 A.D.) contain no new idea in mechanics. For practically twenty centuries no advance was made in the theory of mechanics after the time of Archimedes. "Vir stupendæ sagacitatis, qui prima fundamenta posuit inventionum fere omnium in quibus promovendis ætas nostra gloriatur" is the tribute Wallis penned two thousand years later, when Latin was still the language of scholars and engineers.

The Romans left their mark on civilization as the annals of government, law and language testify, but there does not appear to their credit the discovery of a single scientific principle or the invention of an important mechanical appliance for mitigating the drudgery and toil of mankind. Their slaves and captives labored long and hard, tilling the fields, in the galleys, or with brick, tiles and concrete on the aqueducts.

Every conquest delivered to the Imperial City a new supply of labor. Besides, in tranquil times the legions, kept from mischief by employment on roads, bridges and wall building, supplied abundant labor. In a word the Romans had neither interest in the theory of mechanics, nor the pressing necessity for improved mechanical appliances, as they commanded an abundance of cheap labor. For these reasons this intensely practical people appears to have made no contribution to the science of mechanics.

REFERENCES.

Ball, J. J. The History of Mathematics.
Burr, W. H. Ancient and Modern Engineering.
Darwin, C. Descent, Origin of Species, etc.
Dühring. Geschichte der Principien der Mechanik.
Faraday, M. Proc. Royal Ins. G. B.
Fletcher, B. History of Architecture.
Grote. History of Greece.
Hamlin, A. D. F. History of Architecture.
Heiburg, Leipsic (1881); Heath, Cambridge (1891). Opera Archimedis
Huxley, T. H. Science and Educ., Man's Place in Nature.
Lubbock, J. J. Prehistoric Times.
Mach, E. The Science of Mechanics.
Maspero and Sayce. The Dawn of Civilization.
Perrot and Chipiez. History of Ancient Art.
Petrie, W. M. F. A History of Egypt; Tales; Royal Tombs.
Reber, F. History of Ancient Art.
Renan, E. Mission de Phenicie.
Robinson, J. H. A History of Western Europe.
Rogers, New York (1900). History of Babylonia and Assyria.
Spencer, H. First Principles, Education.
Stark. Archäologie der Kunst.
Tylor. Primitive Culture.
Tylor. The Early History of Mankind.
Tyndall, J. Essays, Notes and Papers.
Winckelmann. Geschichte der Kunst des Alterthums.
Winckler (1900). Die politische Entwicklung Babyloniens und Assyriens.
Wright, D. F. Man and the Glacial Period, etc.

PART II.

I. THE MEDIÆVAL PERIOD, 500–1500 A.D.

1. The Mediæval Attitude toward Science.

The period of societal reconstruction which followed the decay of the Roman Empire was not a time of scientific research or achievement. It was an age of semi-barbarism, tumult and superstititon. Those of gentle and scholarly disposition who sought the quiet asylum of the Church found there a faith in an established cosmography, which did not encourage independent research and investigation of natural phenomena.

Dr. Andrew D. White, of Cornell, says:[1] "The establishment of Christianity, beginning a new evolution of theology, arrested the normal development of physical sciences for fifteen hundred years." This is in part true and it was due, during the first thousand years at least, to a widespread belief, based on the New Testament, that the end of the world was soon at hand. St. Paul had preached: "For ye know perfectly that the day of the Lord so cometh as a thief in the night," and St. Peter had reiterated: "The day of the Lord will come as a thief in the night in the which the heavens shall pass away with a great noise and the elements shall melt with fervent heat and the earth also and the works that are therein shall be burned up."

It was widely proclaimed that the world was in its last days, that just as the antediluvian world was destroyed in the flood, so now the coming of the Lord in a cataclysm of fire was to be awaited from day to day. With such a stupendous supernatural event impending and the termination of the world imminent, devotion to mechanical science was sheer folly. Even such science as had been developed was now become vain and trivial, and was neglected in the face of the duty to watch and to pray.

The end of the world was announced for various specific

[1]"Warfare of Science and Theology," vol. 1, p. 375.

dates notably 1000 A.D., and all endeavor except "saving souls" was pronounced folly and the inspiration of the evil one. And when, after centuries of waiting, the existing order was found going along just as ever, and curious men began to turn again to worldly affairs, they found theology had woven a magic circle and defied any one to find truth outside of it.

In place of verified experience, a literal belief in the Old and New Testament offered a precarious theophany and created a frenzied terror of supernatural agencies. Demons, imps and devils rode the wind and disported themselves to the fevered imagination of the time as the cause of the most common occurrences.[1] Any prying into the secrets of nature was held to be dangerous to body and soul. Physics and chemistry, such as there was, were tabooed as the devil's own arts, and experimental research was anathema.

Stories of interference with the law of gravitation by the devil and the saints are common among the legends of this period. A story published in the Dialogues of St. Gregory the Great, Vol. II, illustrates this belief. During the construction of Monte Cassino about 530, one day the builders found a stone which their united efforts could not move. They reported this to St. Benedict, "who instantly knew the devil was hanging on to it." He exorcised the devil and the stone which before was too heavy for six men became so light that St. Benedict lifted it with ease and put it into the wall. A similar account of the devil increasing the gravity of two marble columns at the Cathedral of St. Virgile, Bishop of Arles, about 600, is given in "Les Petits Bollandistes," Vol. III, p. 162.

Even after the year 1000 A.D., ideas, which to us appear most fantastic, were handed down for generations apparently without anyone doubting their verity or making any endeavor

[1]For the spirit of the time refer to Longfellow's "Christus; a mystery."

> Safe in this Wartburg tower I stand
> Where God hath led me by the hand, . . .
> Safe from the overwhelming blast
> Of the mouths of Hell, that followed me fast,
> And the howling demons of despair
> That hunted me as a beast to his lair.
> (Second interlude.)

to verify them by experiment. When Albertus Magnus (cir. 1250), a famous philosopher of the thirteenth century, believed that the diamond could be softened in the blood of a stag fed on parsley and that a sapphire would drive away boils, it is hardly to be expected that even the learned of this period would have any conception or appreciation of a science of mechanics.

Though St. Paul had advised, "Prove all things, hold fast to what is good," St. Augustine commanded in vigorous Latin —"Major est Scripturæ auctoritas quam omnis humani ingenii capacitas," *i. e.*, accept nothing except on authority of Scripture for that is greater than all the powers of the human mind. When asked, might there not be inhabitants on the other side of the earth, he answered, it is impossible that there should be inhabitants on the other side of the earth, for on judgment day such men could not see the Lord descending through the air. Discussion was closed by authority and debate came to be restricted to such questions as, whether an angel in passing from one spot to another, had to pass through the intervening space.

It came to be considered blasphemous to wish for or to attempt to better earthly conditions, and presumptuous to attempt to explain phenomena except in terms of mystic theology. So, in the course of the centuries, an unfortunate conviction was developed that science was dangerous and evil. This persisted beyond the Reformation. Martin Luther (1483–1546) complained: "The people give ear to an upstart astrologer (Copernicus) who strives to show that the earth revolves,—but Sacred Scripture tells us that Joshua commanded the sun to stand still, not the earth."

In much the same spirit, Melanchthon (1497–1560) declared: "It is the want of honesty and decency to say that the earth revolves and the example is pernicious. It is the part of a good mind to accept the truth as revealed by God and to acquiesce in it." Indeed, theologians of all persuasions, have, at some time, denounced the Copernican idea, for Scripture declares the "sun cometh forth as a bridegroom"—and "the earth standeth fast forever." When a theologian did deign to debate such a topic, his argument was likely to be like that of Fromundus of Antwerp who, in refuting the revolution of the earth, declared that "the buildings would fly off with such rapid motion, and that men would have to be provided with claws like cats to enable them to hold onto the earth's surface."

The theologian who declared in Galileo's time (1600) that "geometry is of the devil," and "that mathematicians should be banished as the authors of all heresies," was but fanatically defending his traditions. The matter is summed up by Huxley in his Essay on Science and Culture (p. 145), where he says: "The business of the philosopher of the middle ages was to deduce from the data furnished by theologians, conclusions in accordance with ecclesiastical decrees. They were allowed the high privilege of showing by logical process how and why that which the Church said was true and must be true and if their demonstrations fell short of or exceeded this limit, the church was maternally ready to check their aberrations; if need be by the secular arm.

Between the two, our ancestors were furnished with a compact and complete criticism of life. They were told how the world began and how it would end; they learned that material existence was a base and insignificant blot on the fair face of the spiritual world and that nature was to all intents and purposes, the playground of the devil; they learned that the earth is the center of the visible universe, and that man is the cynosure of things terrestrial, and more especially was it inculcated that the course of nature had no fixed order but that it could be and constantly was, altered by the agency of innumerable spiritual beings, good and bad, according as they were moved by the deeds and prayers of man. The sum and substance of the whole doctrine was to produce the conviction that the only thing really worth knowing in this world was how to secure that place in a better, which under certain conditions the church promised." There was no place in such a scheme for a science of mechanics.

To the unbiased student there is a measure of truth in the remarks of Dr. White and Dr. Huxley and yet in justice it must be said that they also carry a sting and a reproach which is somewhat unfair. The attitude of the Middle Ages was a growth; it was developed in, and was not imposed on Europe. Whatever reproach there is, should be placed where it belongs on the general ignorance and stupidity of the inhabitants of Europe during this period. There was no conscious conspiracy to retard progress if we except the bigotry, fanaticism and perversion which inevitably accompany ignorance anywhere and in any time. As the general average of intelligence rose, the situation improved. It cannot be denied that the Church was the great conserving and civilizing agency of this era.

All the learning of the ancients was not lost or destroyed outright, but continued to filter through Europe until it gained force under the favorable circumstances of the so-called Renaissance about 1600. But it is most unfortunate that early Christian fanaticism burned and destroyed so much that was good, though "Pagan." In the East, the Greek School at Alexandria preserved, for some centuries A.D., the learning of the ancients, but, about the fifth century, there developed within the church a pronounced spirit of hostility toward the scientific spirit.

Persecutions became common and culminated in 415 A.D. with the murder of Hypatia and the breaking up of the Alexandrian University. In a burst of religious fervor, the schools and a portion of the library were destroyed. The scholars were forced to flee to Byzantium, where schools grew up.

In the west, after 500 A.D., Roman civilization finally went down under the successive inroads of barbarians from the North and culture and refinement were eclipsed in Italy.

Under Constantine (306–337), Christianity was recognized (313) and Byzantium rebuilt as the Eastern capital. Until 476, there were two capitals but the center of wealth and population shifted steadily to this new "City of Constantine." Constantinople soon became the wealthiest and most enlightened city of the world, a quiet retreat for scholarly pursuits. Here many ancient manuscripts on mathematics and mechanics were read, copied, and preserved for posterity.

On the capture of Constantinople, a thousand years later in 1453, this Greek and Byzantine learning was spread to various cities of Western Europe by traveling scholars, and stimulated scholarship in the West. As the mediæval universities grew up from the church schools, their enthusiasm was naturally not in the direction of scientific or mechanical investigation. An age of faith is not inclined to be an age of investigation, and the mediæval period developed little mechanical progress.

The one notable indirect contribution to mechanics in this period was the introduction about 1200, of Hindu arithmetic and Arabic algebra into Europe through the Moors of Spain. Among the ancients, primitive number pictures such as the Egyptian hieroglyphics and the Babylonian cuneiform symbols were used for the digits.

The Greeks used the letters of their alphabet α, β, γ, δ, etc., to represent numbers. The Roman system was little better and no extensive calculation could be performed without the aid of a registering instrument of colored beads called an abacus. Our present powerful system of ten symbols, the "ten digits," and the "method of position," whereby their value depends on their place, seems to have originated with the Hindus, and was carried into Europe by the Arabs.

Leonardo Fibonacci of Pisa (1175) among others is credited with introducing it into Italy by his book Liber Abaci (1202). His introduction reads—"The nine figures of the Hindoos are 9, 8, 7, 6, 5, 4, 3, 2, 1. With these nine and with the sign 0 which in Arabic is called sifr, any number may be written." It is likely that convenience and serviceableness in commerce brought the system into vogue through the trade of Genoa and Venice with the Orient. From these ports the merchants probably spread it by the great overland trade routes through Nuremberg and the Rhine to Antwerp, Bruges and the towns of the Hanseatic League. The money exchanges and the channels of trade probably had more to do with spreading it over Europe than the philosophers.

The college accounts in the English Universities are found to have been kept in Roman numerals up to about the year 1550 and even later. After this date the Arabic system generally displaced the Roman method.

2. The Influence of Moorish Culture.

Before the time of Mohammed (570–632), the Arabs had played an inactive part in history; but, when the wandering tribes of the desert had been welded into a nation by the fiery enthusiasm of the prophet and his fanatical followers, they began to be a factor in the civilization of both East and West.

Within a decade after Mohammed's death, the faith had conquered Arabia, Palestine, Syria and Persia, and within a few years more, the Moslems threatened Europe from the northern coast of Africa. By 711 all Spain, except Asturia, was subject to their sway, and their dominion began to approach in extent the glorious Empire of Rome. In their opinion, the Koran, the new revelation, was destined to supplant the Bible.

The Koran is evidently based on the Hebrew and Christian scriptures. Islam is an offshoot of Christianity, modified to suit the Arabic temperament, by a coloring of Oriental imagery and fatalism. The theological structure of both is the same. There is much the same scheme of rewards and punishments with a tinge of predestination. The prohibition in the Koran against "graven images" was held to forbid the representation of any human or animal form.

This had a marked effect upon their arts, and no doubt encouraged the study of geometry and mathematics in general. On the whole the Moslems seem to have been rather favorably disposed toward pagan culture, regarding it with placid superiority rather than enmity, and they never were hostile to the scientific spirit. When in Europe the practice of medicine was looked at askance, the Arabs were adept in medicine, and their surgeons were in demand in the courts of Europe.

The nomadic Arabs had neither need nor desire for a science of mechanics and the earlier caliphs were too busy establishing their empire, to develop any of the arts and sciences. But toward the end of the eighth century, when their religious fervor was no longer at a white heat, the caliphs became patrons of learning.

Through the Greeks of the conquered provinces, the Mohammedans became acquainted with classical learning and blended it with the wisdom of the Orient, which they had from India and Persia. From the tenth to the thirteenth centuries, the Arabs were the teachers of Europe. They brought about, within their dominions, a renaissance of the Greek culture.

As early as 800, under the caliphate of Haroun-al-Raschid, Bagdad was a famous center of culture. Later, the Western caliphs developed, at Cordova and Seville, schools and libraries which equalled those of Constantinople and Bagdad, and made these Spanish cities famous seats of learning.

In the tenth century, Cordova was one of the greatest centers of commerce of the world and supported eighty schools. The University of Cordova, with its library of 500,000 volumes, became famous throughout Christendom. Philosophy, mathematics, medicine, geography, astronomy and mechanics were taught from Arabian translations of the masters of ancient Greece, Persia, and India.

In working over this material, the Moorish scholars, as was to be expected, developed new ideas and methods, especially in mathematics, astronomy and alchemy. In mechanics and geometry they studied and preserved for posterity the writings of Archimedes, Euclid, and Aristotle. They developed known principles and perfected methods, but it does not appear that they made any very important advance. Extensive fortifications and irrigation works were developed by their engineers, who were well versed in algebra and statics, but had but little grasp of dynamics.

The English champion of Science, Professor Huxley, may be again quoted to advantage on this topic. He says, "Even earlier than the thirteenth century, the development of Moorish civilization in Spain and the great movement of the Crusades had introduced the leaven which, from that day to this, has never ceased to work. At first, through the intermediation of Arabic translations, afterwards by the study of the originals, the western nations of Europe became acquainted with the writings of the ancient philosophers and poets, and in time with the whole of the vast literature of antiquity.

Whatever there was of high intellectual aspiration or dominant capacity in Italy, France, Germany, and England, spent itself for centuries in taking possession of the rich inheritance left by the dead civilizations of Greece and Rome. . . . There was no physical science but that which Greece had created." This found its way into Europe in part through Arabic translations of Greek and Latin texts.

The celebrated Moslem scholar, Al-Khuwarizmi or Mohammed Ibn Musa (cir. 900 A.D.) wrote voluminously on mathematics, on Hindu arithmetic, the sun-dial, and the astrolabe. His "al-jabr-w'al-muqabalah," that is the "redintegration and the comparison," a treatise on algebra, gave the name to this science.

The Arabic numbers and the algebraic method were an immense advance over the clumsy Roman numbers. Without these, it is hardly possible to apply mathematics extensively to mechanical problems. This indicates one reason why the ancients did not advance further in the practical applications of mechanical science. Much advance in mechanics was simply impossible with the old Roman arithmetic which possessed a most awkward duodecimal system of fractions. Decimal fractions date from 1600 when Stevinus, the Flemish engineer, recommended them in his writings.

Of the numerous Christian scholars who attended the Moorish Universities of Cordova and Seville, the most famous was Gerbert, who later became Archbishop of Rheims, and who as Pope Sylvester II (999–1003), exercised a wide influence in Christendom. He is credited with the introduction in Europe of the Arabic mathematics.

The Moors made small original contributions to the science of mechanics, but they are to be credited with the preservation and development of the Greek and Indian knowledge of arithmetic, geometry, and mechanics and the diffusion of it throughout Europe. The Arabic words in our language indicate the breadth of their influence:—algebra, alcohol, Aldebaran, almanac, amalgam, alkali, borax, cipher, carat, minaret, nadir, Vega, zenith, zero. Their invention of algebra and development of the Arabic numerals and notation, while not a contribution to mechanics proper, had a most direct bearing upon the future progress of the science, for without it, the development of our analytical mechanics would probably have been long delayed.

With the coming of the ignorant and fanatical Turks under Genghis Khan in the middle of the thirteenth century, Arabic civilization rapidly declined and the development of mathematics and mechanics was arrested in the Moslem domain. Four hundred years of the Turks has made the once world-renowned Byzantium, one of the most backward cities on the globe. It was not till about 1890 that the Sultan would permit a railway to run into Constantinople.

3. The Period of the Renaissance.

The new order which slowly overcame and displaced the conceptions of mediæval times was the expression of a revolution in the realm of thought. The Renaissance was a period of breaking away from the ideas and ideals of the Middle Ages. It was in part the result of the recognition of certain provinces of thought and endeavor, which the mediæval spirit either ignored or condemned and in part the victory of certain superior features of the civilization of Athens and Rome. The inventions of printing, of gunpowder, of the mariner's compass and the discovery of America, accelerated this tendency, and the religious, political and social changes followed.

With the weakening of the dictates of established authority, men credited personal experience more. They slowly became less biased and more open-minded in their opinions. Pagan writings, which, in mediæval times, were regarded with aversion, if not fear and distrust, came to be studied with interest. Good was found in the manuscripts of the infidel Moslems; their writings were read with interest and appreciation, and their arithmetic was adopted throughout Europe. All this prepared the way for a new start in Science.

Even the theologians began to be dissatisfied with barren dogmas. One of the first to break with the prevailing scholasticism was Cardinal Nicholas of Cusa (d. 1464), who possessed the independence to say that man was prone to err, that it was good to hold one's opinions lightly, and to reject them when they began to appear erroneous. He cultivated mathematics and is said to have taught an imperfect heliocentric theory.

The dawn of the period of the Renaissance may be set at about 1450. Then humanity's native curiosity overcame the terrors of narrow theology. Confidence in the persistence of the order of the universe gained ground and a general interest in the things of the world resulted. The spirit of inquiry soon became rife and with it came a healthy scepticism. In the words of Machiavelli, men began to follow the real truth of things rather than an imaginary view of them.

The mute evidence of cathedral churches left half completed, or with one spire, or none, after the year 1400 or 1500, testifies to the flow of human enthusiasm and energy toward other channels. That so many mighty cathedrals could be constructed in Europe from 900 to 1400 A.D., without advance in the science of mechanics, seems remarkable. But their excellence is in the field of art and not in that of engineering.

Close acquaintance with them reveals to the engineer, poor foundations, cracked arches,[1] crooked walls and leaning towers[2] and settled piers,[3] quite in accord with the annals[4] of failure and collapse which is the history of their construction. In what constituted the spirit of their time, in imagination, in fancy, in inversion of idea, in naïvete of conception, they are

[1]In 1284 the central tower and the apse vaulting of Beauvais Cathedral collapsed utterly. The dome of St. Peter's at Rome would have fallen long since but for the iron bandage of chains placed about the dome in 1742 by Vanvitelli under the direction of Poleni.

[2]The Campanile at Bologna is a well-known example.

[3]Annales de Sevilla, 1677, "On Dec. 28, 1511, a split pillar (of Seville Cathedral) brought down all the central tower and three great arches with a noise that stunned the city. . . . By a miracle of Our Lady of the Sea it did not fall at once. . . . The Archbishop granted indulgences to all who would assist in clearing away the debris." In 1890 it collapsed again.—The utter and complete collapse of the Campanile of San Marco at Venice in 1902 is recent history.

[4]See Hamlin, p. 197, and Feree, Chronology of Cathedral Churches in France.—"The unscientific Romanesque vaulting, etc., resulted in the entire reconstruction of the cathedrals of Bayeux Bayonne, Cambray, Evreux, Laon, Lisieux, Le Mans, Noyon, Poitiers, Senlis, Soissons and Troyes about 1200," etc.

wonderful, but to the trained eye of the engineer, the method of trial and blunder through which they were achieved is apparent.

They are works of art par excellence, there is little science here, except the experienced skill of the master masons, whose closely guarded guild secrets seem to have been trade tricks rather than a science of statics. No evidence has been discovered tending to prove that the cathedral builders had any clear conception of the law of action and reaction, or of the general principle of moments, but they may have used a crude method of determining the ratio of stresses by the funicular method of using weighted strings passing over pulleys. The first formal exposition of this method seems to be in the works of the Flemish engineer Stevinus who was not born till 1548.

The opening of the Renaissance found the science of mechanics not very much further advanced than where Archimedes had left it. Now men began to study and speculate on the subject. Most eminent and successful among those who so occupied themselves are the following:

1. Copernicus (1473–1543), of Thorn in Prussia, who set forth the system of astronomy since identified with his name in "De Revolutionibus Orbium Cœlestium." He maintained that the sun is at rest and that the planets revolve about it, and hinted that theology and mechanics are two distinct branches of knowledge. This quotation dimly presaging the law of gravitation is interesting: "I am of the opinion that gravity is nothing more than a natural tendency implanted in particles by the Divine Master by virtue of which, they collecting together in the shape of a sphere do form their own proper unity and integrity. And it is also to be assumed that this propensity is inherent in the sun, the moon and the other planets."

2. Leonardo da Vinci (1452–1519), the Italian painter, whose manuscripts give a crude idea of the statical moment.

3. Peter Ramus (1515–1572), who contended in his thesis for the Master's degree at the College de Navarre that all that Aristotle taught was false. In his "Animadversiones in

Dialecticam Aristotelis," 1543, he strenuously opposed the scholastic dogmas.

4. Guido Ubaldi, an Italian, who published in his "Mechanicorum Liber" (1577), an imperfect idea of the statical moment.

Of this period is the work of two of the great contributors to the science of mechanics, one in the field of statics, and the other in the field of dynamics, of which he was the founder—Simon Stevinus, an engineer of Bruges (1548–1620), and Galileo, a professor of Florence (1564–1642).

---

4. The Contribution of Simon Stevinus (1548–1620).

Simon Stevinus of Bruges, a military engineer of Prince Maurice of Orange seems to have been a man of genius in experimental research as well as in practical engineering. His earliest extant work is the "Beghinseln der Weegkonst" published in Dutch at Leyden in 1586. The full account of his researches is given in "Hypomnemata Mathematica" (Mathematical Memoranda) a large volume in Latin published at Leyden 1608.

This volume covers in six books, the topics, arithmetic, geometry, cosmography, practical geometry, statics, optics and fortifications. The division on Statics treats of,

1. The elements of statics.
2. The theory of center of gravity.
3. Practical statics.
4. First principles of hydrostatics.
5. Practical hydrostatics.
6. Miscellaneous topics.

This curious medley of theory and practical hints was no doubt the encyclopedia of mathematics and mechanics of the period. A revised edition in French was published by Albert Gerard in 1634. Both editions are very fully illustrated with wood cuts.

We do not find in it any mention of dynamics. Statics is defined as the interpretation of the computations, proportions and conditions of equilibrium (pondus) and of weight (gravitas). The weight of a body is defined as its (potentia descensus in dato loco) force of descent in a given place. Center of gravity is clearly conceived and defined.

Whereas Archimedes considered only the action of parallel forces at right angles to the lever, Stevinus considers the action of forces in any direction and at any angle. He was the first to give a solution of the problem of stability or instability on an inclined plane. His presentation of the simple machines differs from that of Archimedes in that he uses the graphic method of the triangle of forces in the solution of them.

His principal contribution to the science is this idea of the parallelogram or triangle of forces which he gives by many graphical examples without definitely proving it as a general principle at the beginning. It was not completely stated and generally admitted as a principle until about ninety years later when Varignon proved it geometrically and set it forth in a paper before the Paris Academy (1687). In the same year Newton and Lami also published a proof.

It is worthy of note that the first practical exposition of the solution of engineering problems by graphical representation of forces or funicular polygons, now so commonly in use to-day under the name of "graphic statics" was published by engineer Stevinus, about three hundred years ago.

He arrived at the conception of the triangle of forces and the conditions of stability on an inclined plane by his famous "chain of balls on prism" experiment. This is given in the Hypomnemata Mathematica as follows:

II *Theorem*. Proposition 19. If a plane triangle is placed vertically with the base parallel to the horizon, and upon the other two sides are placed single globes in equilibrium then according as the right side of the triangle is to the left so is the balancing effect of the left globe to the counterbalancing effect of the right globe.

Given: Let ABC be the vertical triangle (Fig. 1) with base parallel to the horizon with side AB double BC, and let the globe D on AB be of equal size and weight to that E on BC.

Question: Demonstrate to us that as AB (2) is to BC (1) so the balancing effect of globe E is to the counterbalancing globe D.

Construction. Let us arrange a crown of fourteen balls of equal size and weight strung together at equal intervals, and let there be three fixed points *STV* which are touched by the string so to admit of motion of ascent or descent of the string of balls.

Demonstration: If the balancing effect of the globes *DRQP* is not equal to that of *EF* they must be heavier. Suppose they are, then *ONML* being equal to *GHIK*, the eight globes

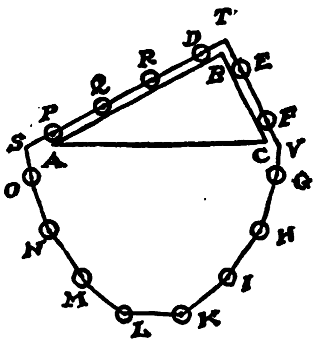

FIG. 4.

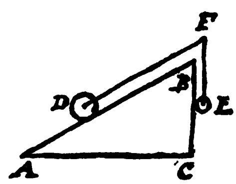

FIG. 5.

(From Liber Statica, Vol. IV, p. 34.)

D, *R*, *Q*, *P*, *O*, *N*, *M* and *L* must overbalance the six *E*, *F*, *G*, *H*, *I* and *K* and the eight will go down and the six will rise up. *D* will go down to *O* and *I* and *K* will take the place of *E* and *F*. But, if this is so, the string of globes will now be situated as before and by the same cause the eight globes on the left will go down and the six on the right will go up, which is saying that the globes of themselves produce continual and eternal motion. This is false. Therefore the part of the string *DRQPNML* holds the part *EFGHIK* in equilibrium. If from equal things equal are taken, equals remain, therefore subtracting *ONML* and *GHIK*, *DRPQ* balances *EF*. But four being held in equilibrium by two, *E* must be doubly as effective as *D*. Therefore as the side *BA* (2) is to the side *BC* (1) so the balancing effect of globe *E* is the counter-balancing effect of globe *D*.

As a corollary it follows that the four balls and the two balls may be concentrated in globes of corresponding magnitudes as indicated in Fig. 2. Or a device like Fig. 3 may be employed.

From this Stevinus comes by corollary to the study of the condition of equilibrium on an inclined plane, which he proves

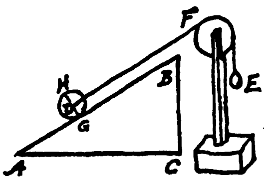

FIG. 6.

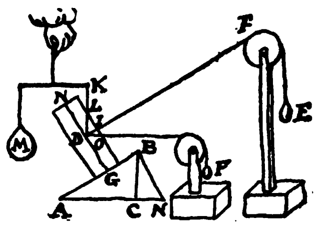

FIG. 7.

(From Liber Statica, Vol. IV, p. 36.)

by the diagram, Fig. 4. This diagram is the earliest exposition of the triangle of forces.

He then generalizes the principle for practical use, in Figs. 5 and 6, where CE is to EO as the weight of the body is to the pull P. From this principle the theory of the funicular polygon is then developed as indicated in Figs. 6 and 7.

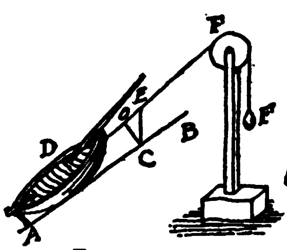

FIG. 8.

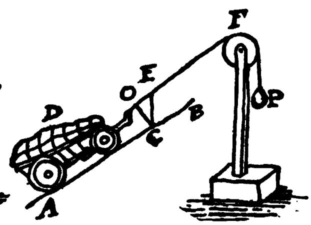

FIG. 9.

From the funicular polygon he advanced to the consideration of the conditions of statical equilibrium in each of the simple machines, referring back to his proof of the inclined plane. Nowhere does he state the principle of the parallelogram of forces explicitly as a general rule from which all cases of equilibrium in machines may be deduced. In the chapter on practical statics the simple machines are fully expounded and their applications indicated in illustrations showing cask being moved into warehouses, etc.

The chapter on hydrostatics is also very practical. The weight of a cubic foot of water at Leyden is noted (62 pounds),

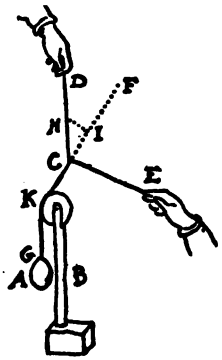

Fig. 10.

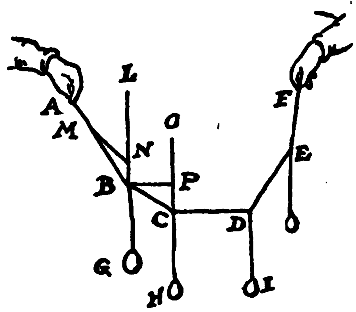

Fig. 11.

(From Liber Statica, Vol. IV, p. 162.)

and suggestions on ship design are given. It is a question as to how much of Archimedes' Hydrostatics was known to Stevinus, but the probability is strong that Stevinus discovered, or at least proved the principle of Archimedes by his

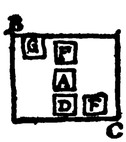

Fig. 12.

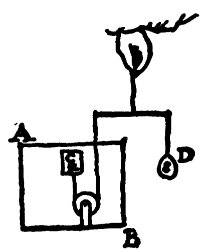

Fig. 13.

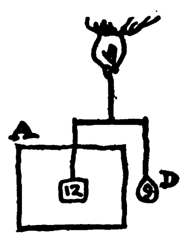

Fig. 14.

(From de Hydrostatices Elementis, p. 119.)

own method. He clearly set forth for the first time the fact that the pressure of a liquid is independent of the shape of the containing vessel and depends upon the height and area of the base. His method of reasoning is simple and convincing and worthy of quotation.

Suppose a mass of water *A* in a jar of still water. This cube is in equilibrium. For, if not, let us suppose it descends, then the water which comes into its place must also descend when it comes into that same place and under similar conditions; but, this leads to perpetual motion which is absurd and contrary to our experiences. Therefore the cube *A* does not move down nor up. It is in equilibrium. If now we suppose the surface of the cube *A* to become solidified, this surface or "vas superficiarium" will be subjected to the same circumstances of pressure.

When it is empty it will suffer an upward pressure equal to the weight of the absent water which balanced the upward pressure. If we fill it with any other substance of any specific gravity it is plain that the loss in weight of that substance in water is equal to this same upward pressure which is equal to the weight of the water displaced. Figs. 9 and 10 illustrate experimental proofs with cubes of specific gravities 1–5 and 4 times that of the fluid.

Granted that the pressure on the base of a cube or vertical parallelopiped of liquid is equal to its weight, by following a similar method of imagining portions of the liquid to become solidified or to be cut out, Stevinus shows that the pressure on the base of a vessel is independent of its form, and proves the laws of pressure of communicating vessels and tubes.

Perusal of Stevinus' notes indicates that he had a hazy idea of the principle of virtual displacements. He had observed that what a simple machine gains in force it loses in distance. In his discussion of pulleys he notes that, "*Ut spatium agentis ad spatium patientis, sic potentia patientis ad potentiam agentis*" (Vol. IV, L. 3) as the space passed over by the force is to the space passed over by the resistance so is the resisting force to applied force. Here he strikes close to the principle of work, namely that in a perfect machine the product of the force and distance traversed is equal to the resistance times the distance through which it is overcome.

We have here in Stevinus' book the germ of the idea of virtual displacements. That is, if in a simple machine we consider any virtual or possible displacement of the agent, the resistance moves over a corresponding displacement so that in every case the product of the acting force and its displacement is equal to the resisting force times its displacement.

Stevinus utilized this idea in a narrow limited way, applying it in the calculations on the simple machines but not attaining to the idea of work, as the measure of force acting through distance, nor to the idea of the balance of positive and negative work in a machine. His chief contributions are the statical principle of the triangle of forces, the founding of Graphic Statics, and the exposition of the conditions of buoyancy and liquid pressure. While he was developing Statics in these directions, his young contemporary Galileo had been experimenting with moving bodies and was laying the foundations of Dynamics.

REFERENCES.

Ennemoser. The History of Magic.
Rydberg. Magic in the Middle Ages.
Döllinger. Studies in European History.
Adams, B. The Law of Civilization and Decay.
Whewell. History of the Inductive Sciences.
Melanchton. Initia Doctrinæ Physicæ.
Bacon. Novum Organum.
Dühring. Geschichte der Mechanik.
Heller. Geschichte der Physik.
Eichen. Mittelalterische Weltanschauung.
Schneider. Geschichte der Alchemie.
Figuier. L'alchimie et les Alchimistes.
Cuvier. Histoire de Sciences Naturelles.
Maury. L'Antiquité et au Moyen Age.
Fahie, J. J. Galileo, his life and work.
Vivian. Life of Galileo.
Boundry, F. Galilee, sa vie.
Lord. Beacon Lights of History.
Mach. The Science of Mechanics.
Ball, J. W. The History of Mathematics.
Cajori, F. A History of Physics.
Alberi. Opere—Geo lettore Galilei, 16 vols.
Mahafy. Des Cartes.
Stevinus. Hyponemata Mathematica.
Nasmith. Pascal.
Gerland & Traumüller. Geschichte der Physical Experimentier Kunst.
Brewster. Martyrs of Science.
Lodge, Sir. O. Pioneers of Science.

5. The Contribution of Galileo Galilei (1564–1642).

Taking up now the work of Galileo we find that he caused a revolution in mental attitude toward the study of natural phenomena. The Aristotelian Natural Philosophy had for centuries been regarded as an infallible authority in the schools. In 1543, Petrus Ramus (1515–1572), a scholar of the University of Paris, was forbidden by an edict of Francis I, under pain of punishment, to teach or write against it. To Galileo, a young medical student of noble Florentine family, who had come to disbelieve in the dogmas of the old philosophy belongs in part the glory of emancipating men's mind from this authority of antiquity.

Galileo appealed from apriori axioms, presuppositions and syllogistic deductions to an investigation of the actual facts. The teachings of Aristotle had been received, "ipse dixit," up to this time, in spite of the fact that some of them were contradicted by daily experience, and in spite of the fact that easy, simple experiments proved them wrong.

To quote but a few of these Aristotelian notions which were blindly accepted and believed—

1. Substances were divided into "corruptible" and "incorruptible," chief among the latter were the heavenly bodies.

2. Bodies were classified as absolute heavy bodies and absolute light bodies and "sought their places"; the light bodies belonging up and the heavy bodies down.

3. Motions were classified as "natural motions" and "violent motions."

4. Large bodies were believed to fall quicker than small ones, or the velocity of falling bodies was believed to be in proportion to their weight.

Galileo vigorously attacked this Aristotelian philosophy; he appealed from authority to experiment, to nature. He boldly contradicted the teachings of Aristotle, which had been accepted and believed for over a thousand years. By direct experiment, as for example, by dropping weights from the leaning tower of Pisa he proved that Aristotle was wrong.

The schoolmen of the time did not readily relinquish their errors and carried on long and bitter controversies with him so that he was obliged to leave Pisa for Padua.

His keenness of perception is well illustrated by the story of the discovery of the isochronism of the pendulum through observations on the gradually decreasing swing of a hanging lamp in Pisa Cathedral. He counted his pulse as the lamp oscillated over a smaller and smaller arc and found that the number remained constant, thus verifying his suspicion of isochronism. He is also credited with the first determination of the relation between the time and length of a pendulum and the application of it in a metronome for the use of physicians. A scheme for a pendulum clock, which he never realized, is found among his manuscripts.

Galileo seems to have been the first to set forth clearly:

1. The idea of force as a mechanical agent.

2. The conception of mechanical invariability of cause and effect.

3. The principle of the independence of action of simultaneous forces.

The rigorous mechanical explanation of motion dates from Galileo. He studied carefully the motions of falling bodies and projectiles and found their laws setting forth:

1. All bodies fall from the same height in equal times.

2. In falling the final velocities are proportional to the times.

3. The spaces fallen through are proportional to the squares of the times.

He came to these laws experimentally by collecting data on the time of descent, the final velocities and the distances traversed, as in the following table, g being a constant.

Time.	Velocity.	Space.
1.	$1g$	$1 \times 1g/2$
2.	$2g$	$2 \times 2g/2$
3.	$3g$	$3 \times 3g/2$
4.	$4g$	$4 \times 4g/2$
t	tg	$t \times tg/2$

The experiment on grooved planes by which these results were obtained are now well known. An inspection of the table shows at once that the numbers follow the simple law,

$$v \text{ varies as } gt,$$

which expresses the relation between the first and second columns,

$$s \text{ varies as } gt^2/2,$$

which expresses the relation between column one and column three, while

$$s \text{ varies as } v^2/2g,$$

is the relation of the second and third columns.

The first two of these expressions, $v \propto gt$ and $s \propto gt^2/2$ were used by Galileo in his development of dynamics to the neglect of $s \propto v^2/2g$. Later, Huygens took up the expression $s \propto v^2/2g$, and made important advances based upon it.

It was observed in the discussions of moving bodies after Galileo's time that a moving body had a certain "efficacy"—that there was inherent in a moving body, something that corresponds to force. Later philosophers debated strenuously as to whether this efficacy was proportional to the velocity or to the velocity squared. But it will be perceived from an inspection of the above expressions that a body with double the velocity can overcome a given force through double the time, but through *four* times the distance. With respect to time, therefore, its efficacy is proportional to velocity; but with respect to distance, or space traversed, its efficacy is proportional to the velocity squared.

Before the time of Galileo, force was treated in mechanics only as pressure; after his time the ideals of force, velocity and acceleration as we know them to-day came into use. That either *acceleration of motion* or *change of shape* is the immediate effect of force is the fact that Galileo perceived and set down as a fundamental and invariable rule of dynamics.

He determines force by the change of velocity, or the acceleration it produces, and he may be said to have discovered the law of inertia indirectly. At all events, his conception of dynamics might be expressed by the formula $F = m.a$, though he did not so express it because his conception of mass was not clear. He made no use of the expression $S = v^2/2g$, which led Huygens to his conception of energy, later formulated as

$F.S = mv^2/2$. His failure to do so appears to be due to the same cause—he did not fully grasp the modern conception of mass.

In Galileo's "Della Scienza Mecanica" (1655), Tom. 1, p. 265, appears the first clear presentation of the principle of virtual velocities. This idea is now common property in several forms, one of which is the familiar dictum, "what is gained in speed is lost in power." It developed slowly into the law of conservation.

In completer form it is now stated as follows: If a material system, acted on by any forces whatever, be in equilibrium; and we conceive the system to experience, consistently with its geometrical relations, any indefinitely small arbitrary displacement; the sum of the forces multiplied each of them by the resolved part parallel to its direction, of the space described by its point of application, will be equal to zero; this resolved part being considered positive when it lies in the direction of its corresponding force, and negative when in an opposite direction.

Though Guido Ubaldi in his "Mechanicorum Liber" called attention to the idea of virtual displacement and moments in connection with the lever, and though Stevinus makes mention of it, Galileo appears to have been the first to apply these ideas to all the simple machines. The term "moment" of a force seems to have meant to Galileo, the effort that tended to set a machine in motion. Therefore, in order that a machine should remain at rest or in equilibrium under the action of two forces it is necessary that their moments balance. He showed that the moments of a force are always proportional to the force times its virtual velocity.

In his "Mechanica, sive de Motu," Wallis uses the term moment in this same sense and bases his statics on the equality of moments as a fundamental principle.

This idea is most prolific and many later writers used it in varied form as the basis of their formal presentation of mechanics. Descartes for example in Lettre 73, Tom. 1, "de Mechanica Tractatus" (1657), bases his whole treatment of Statics on a single principle which is essentially Galileo's idea of virtual velocities. His conception of it is, that it requires exactly the same energy to raise a weight P through an altitude A, as a weight Q through an altitude B, provided that P is to Q as B is to A. It follows from this that any two weights attached to a machine will be in equilibrium when they are disposed in such a way that the small paths they can simultaneously describe are reciprocally as their weights.

The same idea was presented in another aspect by Torricelli in "De Motu Gravium Naturaliter descendentium" (1644). His conception was, that when any two weights rigidly connected together, are so placed that the center of gravity is in the lowest position which it can assume consistently with the geometrical conditions, they will be in equilibrium. Torricelli's principle was finally presented in the form,—any system of heavy bodies will be in equilibrium when their center of gravity is in its lowest or highest position. His presentation was based on Galileo's conception of virtual velocities.

Finally a century later, it was stated about as we have it to-day in general terms by John Bernoulli, in his letter to Varignon dated, Bâle, Jan. 26, 1717, published in "Nouvelle Mecanique," Tom. II, sect. 9.

These ideas gave a new trend to the development of mechanics. In 1743, D'Alembert built the first Treatise on Dynamics on this principle. Had this idea of virtual displacements been clearly perceived and appreciated by Archimedes, mechanics as a science would have developed much more rapidly. This principle is of such universal application that a separate rule is no longer necessary for each of the simple machines; it suffices for them all.

To Bernoulli belongs the credit of showing that the principle of virtual displacements may be made the basis of a whole theory of equilibrium, but the idea originated with Galileo. He also applied the principle in his "Discourse on Floating Bodies" demonstrating by it the theory of buoyancy. In spite of these expositions some of Galileo's opponents still held blindly to the Aristotelian theory that the breadth or form of a body was the factor that determined whether it sank or floated.

In general, Galileo prepared and familiarized men's minds with the correct notion of interdependence of force and motion thus clearing the way for the generalizations of Newton's Laws of Motion. Nowhere does he state these laws explicitly, but their perception is involved in the solution of some of the dynamical problems in his books. He even gives two of the laws of motion in an incomplete way. The first law of Newton is a generalization of Galileo's theory of uniform motion. The second law, that change of motion is due to force and is proportional to the force that makes the change, and takes place in the direction of the force, is a generalization of Galileo's theory of projectile motion.

Before this time, it was commonly believed that a body could not be affected by more than one force at a time, and it was even held that a ball shot horizontally, moved in a straight line until the force was spent and then fell vertically to the earth. Galileo demonstrated in his fourth Dialogue that the path of a projectile must be a parabola, the resultant of a uniform transverse motion and a uniformly accelerated vertical motion.

He, however, did not attain to a clear discrimination between mass and weight, and he failed to see that acceleration might be made a means of measuring the magnitude of the force of gravity. There is no statement of the third law of motion, in reference to action and reaction, anywhere in Galileo's work, though there is a suggestion of the idea of it in some of his statements in the "Della Scienza Mecanica."

Galileo not only founded Dynamics but he made perfectly clear the fact that force may produce two effects upon bodies, change their motion, that is give them *acceleration*, or it may change their form or shape, that is *deform* them. In his study of the first effect he developed the dynamical laws of falling bodies, of projectiles and of the pendulum, in the later he founded the study of the resistance of materials. His crude investigations as to the internal structure of matter and his theory of its deformation and resistance in the form of columns, posts, beams and cantilevers is set forth in his book "Discorsi e dimostrazioni matematiche intorno a due nuove scienze," published in Leyden, 1638. This work attracted little or no attention at the time but it is one of Galileo's most substantial contributions. Although he wrote some sixteen volumes in all, this work and his "Discorso interno alle cose che stanno in sur l'acqua" in which he proves the static law of fluid pressure, contain nearly all his research in mechanics.

After Galileo's time we no longer find such naive and obscure phraseology as, "motions are of two orders, natural and violent." Henceforth the notions of the Aristotelians became untenable. Men soon came to recognize that all bodies, even the heavenly bodies, were probably of one kind. Force came to be understood as that which causes acceleration in a body, or deformation in a body. It became apparent that instantaneous and continuous forces produce unlike effects, and that weight is a continuous force drawing bodies toward the earth. A little later the great Newton explained how it was that all bodies fall with equal velocities from the same height, barring the unequal resistance of the air.

When these things had been pointed out and verified by experiment, the foundation of the study of moving bodies was laid and the progress of Dynamics was sure and steady. Newton's generalizations followed logically upon Galileo's discussions of motion, and D'Alembert's Treatise on Dynamics came as an expansion of these ideas. It is no exaggeration to say that to Galileo we owe modern mechanics. Lagrange in his "Mecanique Analytique" testifies to Galileo's greatness in these words:

"Dynamics is the science of forces accelerating or retarding, and of the various movements which these forces can produce. This science is entirely due to moderns, and Galileo is the one who laid its foundations. Before him philosophers considered the forces which act on bodies in a state of equilibrium only; and although they could only attribute in a vague way the acceleration of heavy bodies, and the curvilinear movements of projectiles, to the constant action of gravity, nobody had yet succeeded in determining the laws of these daily phenomena on the basis of a cause so simple. Galileo made the first important steps, and thereby opened a way, new and immense, to the advance of mechanics as a Science.

"These discoveries did not bring to him while living as much celebrity as those which he had made in the heavens; but to-day his work in mechanics forms the most solid and most real part of the glory of this great man. The discovery of Jupiter's satellites, of the phases of Venus, of the Sun-spots, etc., required only a telescope and assiduity; but it required an extraordinary genius to unravel the laws of nature in phenomena which one has always under the eye, but the explanation of which, nevertheless, had always baffled the researches of philosophers."

PART III.

THE MODERN PERIOD, 1500 TO 1900.

We no longer believe with the cave-men that thunder is the roar of an angry god, nor with Luther that a stone thrown into a pond will cause a dreadful storm because of the wrath of devils kept in prison there; but we still believe with them that wood floats, and we have clear ideas of the conditions of its buoyancy. The characteristics of the modern period are its empiricism, the great and increasing part played by natural knowledge, and a strong conviction of the importance of sense impressions as a source of knowledge. Added to this we observe an enthusiasm for research and a determination to expose error regardless of controversy or consequences.

Since 1700 the whole outlook upon the universe has changed. Science has routed the old theology, and altered the habits of life of millions by its influence in the trades and industries. Though there is still nothing more mysterious than force, the imps of that weird ante-world of Science that lurked in every zephyr and grinned from every tree and dark nook are now no more. Quite apart from the comforts of life that we owe to mechanics, we are indebted to the science for the peace of mind which a rational Natural Philosophy has brought us.

Since Galileo's time mechanics has been characterized by an attitude of direct experimental inquiry which has sought to test and extend the conceptions already formed. The science has grown by slow expansion and accretion, and it has often been some time before new conceptions have become susceptible of precise statement. It seems as though an idea must be considered and turned over by many minds before it can be clearly set forth. Even the great masters have not always presented the principles which they have contributed, in the form in which we now state them.

Thus it is, that a clear statement of the ideas developed in this later period is not attained to, until a number of workers have cultivated the field, and until each has developed separately facts, which when correlated and worked over, by a master mind, give us an illuminating view of the whole subject.

So, in passing from the work of Galileo to that of the next great master Christian Huygens (1629–1696), we pass over a number of men whose work was one of preparation, extension and amplification, rather than of new contribution. Such were these workers whose activities can be but briefly referred to in this outline of the history of mechanics.

Johann Kepler, 1571–1630, of Würtemburg, Germany, who in "Astronomia Nova," 1609, and "Harmonice Mundi," 1619, set forth the three laws of planetary motion, viz:

(1). Each planet revolves in an elliptic orbit having the sun as its focus; (2) the straight line joining the sun and planet passes over equal areas in equal times; (3) the square of the time of revolution of each planet is proportional to the cube of its mean distance from the sun. Here we have the statement that the solar system is disposed according to mathematical and mechanical law.

Francis Bacon (1561–1626), author of Novum Organum (1620), who declared for science on the ground that "knowledge is power" and who advocated an experimental study of the world with a view to improving human conditions.

Marcus Marci, 1595–1667, published at Prague in 1639 "De Proportione Motus" in which he gives correct elementary notions of impact.

René Descartes, 1596–1650, published at Amsterdam in 1644 his "Principia Philosophiæ" in which we have the first notable modern endeavor to formulate a system of mechanics from the universal point of view. His scheme is objectionable, but he called attention to the problem of a universal mechanical philosophy.

Gilles Personne de Roberval, 1602–1675, published in the Memoirs of the French academy, 1668, a notable paper "Sur la Composition de Movements."

Otto Von Guericke, 1602–1686, of Magdeburg, Germany, an engineer in the army of Gustavus Adolphus published "De Vacuo Spatio," 1663, and "Experimenta Nova," 1672, giving an account of his invention of the air-pump, 1650, and various experiments performed with it.

Pierre de Fermat, 1601–1665, of Montauban, France, published between 1670 and 1680 a series of monographs on maxima and minima, tangents, curves, centers of gravity and copies of his correspondence with Descartes, Huygens and Pascal on mechanical problems, under the title "Opera Mathematica," which cleared the way for later advance.

Evangelista Torricelli, 1608–1647, constructed the first mercurial barometer about 1643 and applied it to the measurement of variations in atmospheric pressure.

Edme Mariotte, c. 1620–1684, who in his "Traité du Mouvement des Eaux," 1686, published the first treatise on hydraulics and advocated and developed experimental research on gravitation, hydraulics and pneumatics.

Robert Boyle, 1627–1691, who first formulated what is now known as "Boyle's Law," viz: that when a gas is at a constant temperature, the product of the pressure and volume remains constant, howsoever one of these be varied.

Blaise Pascal, 1623–1662, who published his studies on the question of fluid pressure in "Recit de la grande experience de l'equilibre des liquers" (1648), and "Traite de l'equilibre des liquers et de la presanteur de la masse de l'air" (1662). One of his conclusions, commonly known as Pascal's principle is, that "external pressure is transmitted by fluids in all directions without change in the intensity."

All of these investigators either set forth an idea of some importance or simplified and extended the presentation of accepted ideas. Later investigators were familiar with their work and mounting upon it attained to the higher conceptions of the science. The work of Kepler in astronomy was particularly useful to Newton in attaining to his grand generalization of the law of universal gravitation. The work of Descartes besides presenting a highly useful method of combined algebraic and geometric analysis, suggested the idea of Conservation, while the branches of hydrostatics and pneumatics could not have progressed far without the researches of Guericke, Torricelli, Mariotte, Pascal and Boyle. Their work was one of investigation, experiment and study in rather narrow fields. It was a work of preparation upon which their successors built grandly.

1. The Contribution of Christian Huygens (1629–1696).

Most of the great masters of mechanics have left a record of practical invention as well as of theoretical advance in the science. Huygens is remembered as the man who first made a good clock. Born at The Hague, and educated at Leyden and Breda, where he studied law and mathematics, he won fame in astronomy as well as in mechanics. In 1665 he discovered the rings of Saturn with a telescope which he had constructed. We are concerned however only with his contribution to mechanics. He carried forward Dynamics by developing precise statements of accelerated motion and solving the first problems in the dynamics of several masses. Galileo had always restricted his speculations to a single body.

Huygens' contributions are set forth in his publications;—"A summary account of the laws of motion," Philosophical Transactions, 1669, "Horologium Oscillatorium," Paris, 1673, and "Opuscula Posthuma," Leyden, 1703. The complete mathematical theory of the pendulum and his invention of the escapement is completely set forth in the "Horologium" (1673), which is a work worthy to rank with Newton's Principia. He was the first to determine the acceleration of gravity by the pendulum, and also the first to enunciate the formula of centrifugal force, $F=mv^2/r$; his discovery of the laws of collision of elastic bodies was announced simultaneously with that of Wallis and Wren.

Huygens' great work was a complete exhaustive theory of the pendulum and the solution of the problem of center of oscillation. He considered the pendulum to be made of particles, and originated the idea of the dynamics of more than one mass or particle. He employed the method of aggregating the motions of particles, but though he used symbols for "moment of inertia" and "statical moment," he did not define and assign these names. Euler appears to be responsible for the term moment of inertia ($\Sigma m.r^2$). This method, now so general in its application in the mechanics of solids and liquids and in daily use by engineers and physicists, is one of the great inventions of Huygens. It gave to the science of mechanics a new trend, and when the invention of the calculus made the process of summation easy, this method brought a wealth of progress in dynamics and hydrodynamics.

In the dispute as to whether the so-called "efficacy" of a moving body is proportional to the first or second power of the velocity, Huygens, who had originated the later idea, maintained it strenuously. The dispute was really one of terms. The "efficacy" of a moving body varies as its velocity in reference to the time and as the square of the velocity in reference to the space passed over. Reference to the time leads to what Descartes called the "quantity of motion" (momentum), $m.v$. This makes the notion of force the primary concept. Reference to the distance passed over gives the expression, $m.v^2$, which makes work or energy the primary concept. The first view is expressed by $F.t = m.v$ as the fundamental equation of mechanics, the second gives: $F.s = m.v^2$ as the fundamental equation.

In 1847, Belanger proposed the name impulse for the expression $F.t$, which was later adopted and popularized by Clerk Maxwell in his writing on matter and motion.

Leibnitz gave the name "vis viva" to the expression $m.v^2$ in a memoir published in the "Acta Eruditorum," 1695, entitled "Specimen dynamicum pro admirandis naturæ legibus circa corporum vires et mutuas actiones detengendis et ad suas causas revocandis," or "A dynamic illustration of the astonishing laws of the power of bodies in their reciprocal action revealed and traced back to their causes." He intended, as the name visa viva suggest, to indicate a measure of the force of a body in actual motion. The term "vis motrix" was also used interchangeably with vis viva to distinguish the moving force from statical pressure which was called "vis mortua."

The difference of opinion really hinged on whether force or energy is to be considered the fundamental notion. Huygens and his party maintained the latter position, while Descartes and later Newton accepted force, mass and momentum as fundamental notions. This dispute went on for fifty years until D'Alembert in 1740 in his "Dynamique" showed it to be a misunderstanding as to terms, not facts.

Later Coriolis (1792–1843) introduced the more common notation of $\frac{1}{2}m.v^2$ for vis viva or kinetic energy, and Poncelet adopted the same plan, but the conception of energy we owe then to Huygens.

Another of his important achievements was the solution of the problem of center of oscillation. This cannot be done without recourse to the new method which he used so successfully, namely the method of the dynamics of particles. It is a matter of every-day observation that a long pendulum oscillates more slowly than a shorter one. Therefore if we consider the component particles of a compound pendulum as so many simple pendulums, it is manifest that owing to their connections they all vibrate with only one determinate period of oscillation. There must exist a simple pendulum that has the same time of oscillation as the compound pendulum. Its length measured off on the compound pendulum gives us the particle or point that preserves the same period of oscillation as if it were detached and vibrating as a simple pendulum. This point is the center of oscillation.

The idea which Huygens applied in the solution of this problem is, that in whatever manner the particles of the pendulum may by their mutual interaction modify each other's motions, in every case the velocities acquired in the descent of the pendulum will be such only that the center gravity of the particles, whether still in connection or with their connections dissolved, is able to rise to the same height as that from which it fell.

Huygens' proof in brief is:—Let OK be a linear pendulum made up of a large number of masses set in a line OK. If it be set free it will swing through B to OK' where $KX = XK'$.

The center of gravity will ascend just as high on the second side as it fell on the first. Now if at OX we should suddenly release the individual masses from their connections the masses could by virtue of the velocities impressed upon them by their connections, only attain the same height with respect to center

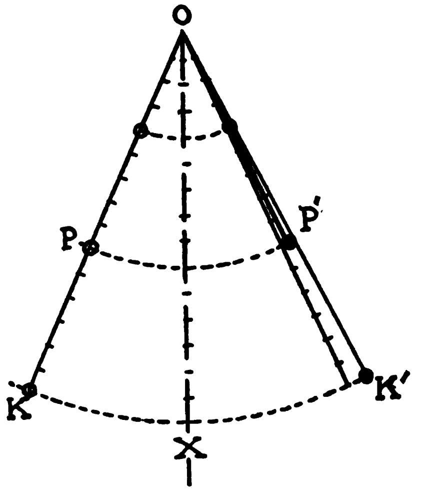

FIG. 15.

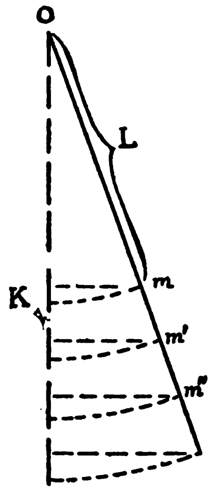

FIG. 16.

of gravity. If the free outward-swinging masses be arrested at the greatest heights they attain, the shorter pendulums will be found below the line OK', the longer ones will have passed beyond it, but the center of gravity of the system will be found on OK' in its former position.

The enforced velocities are proportional to the distances from the axis; therefore, one being given all are determined, and the height of ascent of the center of gravity may be found. Conversely the velocity of any particle is determined by the known height of the center of gravity. So if we know in a pendulum the velocity corresponding to a given distance of descent we know its motion is defined.]

If now on a compound linear pendulum we cut off the portion L equal to l, and if the pendulum move from its position of greatest displacement to the position of equilibrium, the point at the distance l from the axis falls through the distance K. The masses m, m', m'' at the distances r, r', r'', will fall the distances rk, $r'k$, $r''k$ ... and the distance of descent of the center of gravity will be:

$$\frac{mrk+m'r'k+m''r''k+\cdots}{m+m'+m''+\cdots}=k\frac{\Sigma mr}{\Sigma m}.$$

If now, the point at the distance l acquires on passing through the point of Equilibrium a velocity v, the height of ascent assuming the dissolution of connections will be $v^2/2g$ and the heights of the other particles will be $(rv)^2/2g$, $(r'v)^2/2g$, $(r''v)^2/2g$ . . . and the height of ascent of the center of gravity of the liberated masses will be:

$$\frac{m\dfrac{(rv)^2}{2g}+m'\dfrac{(r'v)^2}{2g}+\dfrac{m''(r''v)^2}{2g}+\cdots}{m+m'+m''}=\frac{v^2\Sigma mr^2}{2g\Sigma m},$$

and

$$k\frac{\Sigma mr}{\Sigma m}=\frac{v^2\Sigma mr^2}{2g\Sigma m}. \qquad (a)$$

But, to find the length of the simple pendulum that has the same period of oscillation as the compound pendulum, it is necessary that the same relation must exist between the distance of its descent and its velocity as in the case of unimpeded fall. If y is the length of this pendulum, ky is the distance of its descent and vy its velocity. Therefore,

$$\frac{vy^2}{2g}=ky$$

or,

$$y\cdot\frac{v^2}{2g}=k. \qquad (b)$$

Multiplying equation (a) by equation (b) we get

$$y=\frac{\Sigma mr^2}{\Sigma mr}.$$

Here we note Huygens' recognition of work as the determinative of velocity, and we see that he measures it in terms of the second power of the velocity, *i. e.*, V^2. This work he called the vis viva of any system of masses such as m, m', m'', having the velocities v, v', v'' as expressed in the formula.

$$\frac{mv^2}{2}+\frac{m'v'^2}{2}+\frac{m''v''^2}{2}+\cdots$$

Huygens thus clearly indicated that the center of gravity is conserved or cannot rise higher than it falls and in establishing it he sets forth what he calls the principle of vis viva, or the rule that the work or energy is proportional to the mass times the velocity squared. The equation for work and energy $F.s=m.v^2$ is really an algebraic statement of Newton's second law of motion and is fundamental in dynamics. Lagrange based his work on it.

The writers before Huygens did not have a clear conception of mass as distinguished from weight nor was he perfectly clear on this point, but his endeavors to explain the error of the pendulum clock of the Jean Richer expedition to Cayenne (1671), by the greater centrifugal force at the equator, show that he had the idea of mass as somehow different from weight.

Huygens may then be said to have contributed the mechanical principles symbolized by those type-expressions and their simple derivatives,

(1) $$\Sigma mr^2,$$

(2) $$F=mv^2/r,$$

(3) $$F.s\propto m.v^2.$$

Thus Huygens originated the mathematical method by which the ideas of Galileo were applied to a variety of problems. The development and amplification of these contributions by their successors brought a wealth of progress. Huygens' general way of attacking problems of masses under the action of forces by the method of the *dynamics of a particle* is in almost daily use by the physicist and the engineer. Like some of the contributions of Archimedes, the contributions of this great master Huygens have an eternal value.

REFERENCES—HUYGENS.

Œuvres completes de Christian Huygens, 6 vols. (1888 La Haye).
Christiani Huygenii de circuli magnitude inventa.
Christiani Huygenii Zuilichemii Opera Reliqua (Amsterdam, 1728).
Christiani Huygenii a Zulichem Opera Varia, 1724.
Christiani Huygenii Zulichemii Opera Mechanica; Geometrica, Astronomica et miscellanea, 1751.
Christiani Huygenii Kosmotheros, 1698.

2. The Contribution of Sir Isaac Newton (1642–1727).

The year of Galileo's death was the year of Newton's birth. With the passing of the great Italian scientist, a worthy successor, the greatest of all experimental philosophers, was born in England. He established order in the domain of Science and set forth the great laws by and through which mechanics has been able to grow and prosper.

Newton's contribution may be considered under two headings: the development of dynamics, and the applications of dynamics to the great problem of planetary motions. His work is a logical sequence to that of Stevinus, Galileo, Kepler and Huygens. The "Philosophiæ Naturalis Principia Mathematica" of Newton (1687), commonly called the Principia, is one of the most extraordinary products of human genius, not only in itself but in the revolution which it effected in theoretical and practical mechanics. In it we find much more than a re-statement of the general principles of equilibrium, center of gravity and mechanical powers which were common property at this time. It is a body of doctrine based upon the contributions of all preceding inquirers reduced to the lowest terms.

The whole body of doctrine on motion of projectiles which had been developed by Galileo, Huygens and others, is reduced to the concise, comprehensive "axioms" or laws of motion. We have then in the very opening pages of the Principia, a clarification, precipitation and crystallization of all previous contributions. This of itself was a great gain, but it was done by Newton as preliminary to further advance, preliminary to dynamical discussions which in their grand scope, sweep from the earth to the planets and beyond to the utmost limits of the universe.

The "composition of forces" he indicates as a corollary to the laws of motion. This was a new idea. In Newton's point of view if a body in space is acted upon by an impulsive force it moves in a straight line. If at the same time, another force acts upon it at an angle inclined to the first force, the body takes an intermediate course called a "resultant" path determined by drawing the diagonal of the parallelogram, the two sides of which represent the magnitude of the two forces.

The truth of this principle is made to depend upon the laws of motion. From being a mere statement of experience, as it was with Galileo and Stevinus the parallelogram of forces is correlated to the fundamental laws of motion and deduced as following at once from them. What Galileo and Stevinus said in pages, Newton said in a paragraph. The principle of the parallelogram of forces is not merely stated, it is deduced from three fundamental laws of motion.

On the same axioms Newton based the whole theory of central forces. He supposes a body to be acted upon by two forces as above, but supposes that the second force acts on it in a new direction in succeeding instants. Then the successive diagonals of the parallelograms of the forces will be successive sides of a polygonal figure and the lines of the deflecting forces will cut one another within the figure. If they meet in a point forming a series of triangles of equal area it is easy to see that the path of the body is the same as though a single force acted upon the body to produce motion forever in a straight line and a second force acted upon it to deflect it continually to a point within the polygon. The limit of such a path, as the polygonal sides become smaller, is a curve.

In this manner centrifugal forces and curvilinear motion are demonstrated and their laws set forth. Thus a whole system of dynamics is developed from the geometrical and mathematical relations of diagrams of the parallelogram of forces. Newton founded the correct theory of motion about a center and the whole system of dynamics involved with it.

He set forth the principle of equal areas described in equal times as the test of a central force and gave the mathematical proof.

A body exposed to the action of a central force and given an impulse in a straight line will neither fall toward the central force nor proceed in a straight line, but will take an intermediate diagonal or curvilinear path. Newton brought to bear on this problem all the geometrical and mathematical knowledge of his day, as well as his own fluxional calculus, and established the theorem that a body projected in a straight line and subjected to the action of a central force will revolve in some one of the conic sections if the force vary inversely as the square of the distance from the focus,—which of the conic sections, he shows, depends on the ratio of the forces. This dynamical theorem is the starting point of Newton's system of celestial mechanics.

A variety of consequences follows mathematically from this theorem. The dynamics of elliptic orbits is established at once upon a sound basis and the interrelation of the functions of motion follow inevitably.

He also took up the abstract theory of the attractions which portions of matter may be conceived to exert upon each other, showing that if the particles be attracted according to the law of inverse square of the distance and if they be aggregated into spherical masses these spheres will themselves attract accordingly to the same law, and that the attraction would be directed to the centers of the spheres and be proportional to the matter contained in them, divided by the square of the distance between the centers.

From this, to the law of universal gravitation seems but a step. But though Newton is said to have entertained this theory as early as 1666, it was not till 1672, when the data of Picard on the figure of the earth were obtained, that Newton justified it and became convinced of its truth. He first gave it out in his lectures in 1684. It was published latter in his treatise "De Motu" and in the "Principia." In Book III, Proposition 4 of the latter, he calculates the acceleration of the moon toward the earth and shows that starting from rest with this acceleration, it would fall towards the earth 16 feet in the first minute and that at the earth's surface 60 times nearer, the same distance would here be fallen through in one second which was almost exactly the value obtained by Huygens in his experiments.

The action of force without a medium to transmit it, appears to have troubled Newton, and this is not surprising when one considers that the pull of the sun on the earth is equal to a force sufficient to break a million million round steel rods each twenty-five feet in diameter.

The law of universal gravitation is the basic principle of Newton's applications of his dynamics to planetary motions; when he had achieved it, all the mechanism of the universe lay like an open book before him. It was now possible to apply mathematical analysis with absolute precision to the problems of astronomy.

At once many new conceptions came into view. Neither Galileo nor Huygens had clearly distinguished *mass* from *weight*, but now it followed at once, that the same body must have a different weight at different places on the surface of the earth, and might even be conceived under certain conditions to have no weight. We arrive now for the first time at a clear idea of mass. The idea of force as first propounded by Galileo was now seen to be of universal application. Finally the law of action and reaction was clearly stated and set forth. These are most illuminating conceptions.

It began to be evident after this, that gravity was a force measurable like any force in terms of mass and acceleration, though it was some time before the principle was stated in the concise algebraic form,

$$F = m.a,$$
$$W = m.g,$$

Newton's concept of mass is in fact the corner-stone of his dynamics.

At the beginning of the Principia we find a series of fundamental conceptions given in a series of definitions. The final one refers to mass, as follows:

Definition I. (As to mass.) "The quantity of any matter is the measure of it by its density and volume conjointly. This quantity is what I shall understand by *mass* of a body in the discussion below. It is ascertainable from the weight of the body for I have found by pendulum experiments of high precision, that the mass of a body is proportional to its weight, as will hereafter be shown.

Definition II. "Quantity of Motion" is the measure of it by its velocity and quantity of matter conjointly.

Definition III. The resident force "vis insita," *i. e.*, inertia of matter is a power of resisting, by which every body, so far as in it lies, persists in its state of rest or of motion in a straight line.

Definition IV. "An impressed force is any action which changes or tends to change the state of rest or of uniform motion in a straight line." This defines force as the cause of acceleration or tendency to acceleration of a body.

The laws of motion as Newton enunciates them are:

Law I. Every body persists in its state of rest or of uniform motion in a straight line, except in so far as it is compelled to change that state by impressed forces.

Law II. Change of motion (*i. e.*, of momentum) is proportional to the moving force impressed, and takes place in the direction of the straight line in which such is impressed.

Law III. "Reaction is always equal and opposite to action, that is to say the actions of two bodies upon each other are always equal and directly opposite."

To these are added a number of corollaries. The first and second relate to the principle of the parallelogram of forces, the others are logical consequences of the laws. Then follow the propositions in two books, the first treating of the motion of bodies in non-resisting media and the second in resisting media.

The work of Newton may be summed up in his definitions and laws. The great result of his work was the clear concept of mass and the conception that bodies mutually cause acceleration in each other dependent upon space and material circumstances.

Mach[1] sums the matter up by saying that in reality only *one* great fact was established, viz: that "different pairs of bodies determine independently of each other, and mutually, in themselves, pairs of accelerations whose terms exhibit a constant ratio, the criterion and characteristic of each pair."

Newton did not state his results in algebraic terms. He developed the deductions in the "Principia" geometrically though there seems to be no doubt but that many of his conclusions were arrived at, and perhaps first proven by his method of fluxions or fluxional calculus.

Had Newton stated his principles algebraically, the formula for force, $F=m.a$ would probably have been expressed by him as $F=m.v/t$ or $F.t=m.v$. Acceleration he would have expressed as the time rate of change of velocity v/t, and forces he would have measured by their momentum, $m \times v$. Up to Newton's time such formal presentation of the Science of Mechanics as there was, had been made on the geometrical method and Newton following in the steps of Archimedes, Stevinus and Galileo, used this method in his "Principia," representing forces by lines by the graphical method.

The fact that acceleration may also be expressed as a space rate of velocity squared, or analytically by the formula $a=v^2/2s$ is not apparent geometrically. Substitution in the algebraic formula $F=m.a$ gives $F.s=m.v^2/2$, or the expression for work and energy which is not considered in Newton's geometrical analysis, and which was the point of view from which Descartes, Huygens and Leibnitz approached the subject.

This difference in point of view gave rise to the long controversy already mentioned between the English disciples of Newton and the continental school or the adherents of Huygens and Leibnitz. The so-called Galileo-Newtonian school maintaining that momentum ($F.t=m.v$) was the only correct measure of force and the Leibnitzian-Huygenian school maintained with equal vigor that force was a function of the "vis-viva" or energy $\left(F \cdot s = \frac{m \cdot v^2}{2}\right)$.

[1]Dr. E. Mach, Science of Mechanics.

For fifty years mechanics developed along these two separate paths. The English investigators long followed the formal geometrical presentation of Newton, and the French, Germans and Swiss developed their mechanics on the work of Huygens, using the calculus of Leibnitz. It was not till mathematical analysis came to be applied to mechanics and an analytical scheme was developed by D'Alembert in his "Traite de Dynamique" (1743) that they were reconciled, and brought into accord.

It was then seen that both can be derived from Newton's fundamental equation $F=m.a$. If the acceleration is measured as a time rate of velocity, we get $a=v/t$ and if the acceleration is measured as a space rate of the velocity squared we get $a=v^2/s$, which are of the same dimensions.

Newton made very clear the conception that the effect of a force is to change the size or shape of a body or to change its velocity, that is to give it acceleration. For studying the flux or flowing relations of quantities he devised his "Method of Fluxions" now commonly known as the calculus. The basic idea of his Fluxions is this. He considers a "fluent" as a quantity that gradually and indefinitely increases or flows. The velocities at which such fluents move he defines as fluxions ("Quas Velocitates appello Fluxiones, aut simpliciter Velocitates vel Celeritates"). With the development of this method we have at hand an instrument for tracing changing phenomena by the relation or ratio of elements. This is an invention of inexpressible value to Mechanics.[1] The method appears to have been first used by Newton as early as 1666 and is found in his MS. "De Analysi per Equationes Numero Terminorum Infinitas," which was given to his students in 1669.

Sir Isaac Newton is to be credited then with a general clarification and formulation of the investigations of all his predecessors, and these specific contributions:

1. The concept of mass.
2. The generalization of the idea of force.

[1]Prof. John Perry's "Calculus for Engineers" exemplifies the practical value of the Calculus of Newton.

3. The laws of motion.
4. The theory of central forces.
5. The theory of attraction.
6. The system of dynamics based on the conception $F.t = M.v$.
7. The method of fluxions or the fluxional calculus.
8. The law of universal gravitation, and the application of his abstract dynamics to planetary motions.

Before Newton the science consisted of the more apparent rules of statics as developed by Archimedes and Stevinus and the uncorrelated principles of dynamics as worked out by Galileo and Huygens. Newton reduced these cumbersome unconnected rules of statics and dynamics to three formal laws of motion and founded a system of dynamics of universal application which has been found all-sufficient to co-ordinate the mechanical phenomena of the universe.

In conclusion we may say that the principles formulated by Newton cover all statical and dynamical problems. Much of the work of later masters has been a verification and an extension of the work begun by him. The scienc of mechanics, as now generally taught, is founded upon them.

Playfair says[1] in his dissertation on Newton, "No one ever left knowledge in a state so different from that in which he found it. Men were instructed not only in new truths, but in new methods of discovering truth; they were made acquainted with the great principle which connects together the most distant regions of space as well as the most remote periods of duration and which was to lead to future discoveries, far beyond what the wisest or most sanguine could anticipate."

REFERENCES—NEWTON.

Brewster, D. Memoirs of Sir Isaac Newton. (2 vols.)
Ball, W. R. An essay on Newton's Principia.
Newton. Principia.
Mach, E. The Science of Mechanics.
Ball, W. R. A Short History of Mathematics.
Cittenden. Life of Newton.

[1]Page 133, Dissertation 11, Playfair's "Progress of Math. and Phys. Sciences," 1820.

Hosley, Opera Omnia, 1779.
Maxwell. Matter and Motion.
Thomson & Trait. Natural Philosophy.
Cajori, F. A History of Physics.
Hersley's Newton. London, 1785.
Pearson, C. The Grammar of Science.
Pemberton. View of Newton's Philosophy.
Playfair. Progress of Mathematical and Physical Sciences, 1820.
Perry, J. Calculus for Engineers.

3. The Contributions of Varignon, Leibnitz, the Bernoullis, Euler and D'Alembert.

Pierre Varignon (1654–1722).

In the same year that Newton published his Principia (1687), there was presented before the Paris Academy a work on Statics by Pierre Varignon based on the principle of moments which he developed geometrically from the parallelogram of forces.

The book was published after his death under the title, "Project d'une Nouvelle Mecanique" with the dedication, "Illustrissimo clarrissimoque viro D. D. Isaaco Newton." It begins, "La Mecanique en general est la Science du Mouvement, de la cause de ses effects; en un mot de tout ce qui y a rapport. Par consequent elle est aussi la science de proprietez et des usuages de Machines ou Instruments propres a faciliter le mouvement;" *i. e.*, "Mechanics is in general the science of motion, of its cause and of its effects; in a word of all that pertains to motion. Consequently it is also the science of machines." We meet here in Varignon's book a system of mechanics which is essentially dynamical, including statics as the special case where forces counterbalance.

After defining mechanics thus, as the science of motion and the theory of machines, he says this treatise will be divided into ten sections:

(1) Axioms, postulates and propositions;
(2) Weights suspended or supported by strings;
(3) Pulleys;
(4) Wheel and axle;
(5) The lever;

(6) The inclined plane;
(7) The screw;
(8) The wedge;
(9) The general principle of the simple machines;
(10) The equilibrium of fluids.

The first section treats of the definitions, axioms and hypotheses upon which the work is based. The idea that forces may act each upon other and maintain a body at rest is emphasized. Then the suppositions are made that in the geometrical treatment of machines the parts are to be considered as without weight and friction, perfectly mobile upon their axes, cords are to be considered as perfectly flexible without weight, without elasticity and without stretch or elongation.

The principle of the parallelogram of velocities is now stated geometrically as,

Lemma I.

In order to help the mind to conceive compounded motions let us conceive the point A without weight to move toward B along the line AB, and at the same time suppose that the line itself moves uniformly towards CD along AC remaining always parallel to itself, that is to say maintaining always the same angle with AC. Of these two movements commencing at the same time let the velocity of the first and of the second be as the sides AB and AC of the parallelogram $ABCD$. Then in the parallelogram, I say that by the action of the two forces upon A, this point will travel along the diagonal AD of the parallelogram during the time that AB and AC are being traversed.

Lemma II.

If the point A without weight is pushed in the same time and uniformly by two forces E and F acting upon it, along the lines AC and AB acting at the angle CAB. The united action of these two forces will move A along the diagonal of the parallelogram AD in the same time that A would move to C or to B and as though a force in the proportion of AD to CA or AB had acted upon it.

The parallelogram of velocities and of forces is here conceived as axiomatic. On these conceptions Varignon builds up a logical geometrical development of mechanics. He demonstrates the principle of statical moments by a geometrical theorem in which he shows that the product of a force F (represented graphically by a line), times its lever arm, a

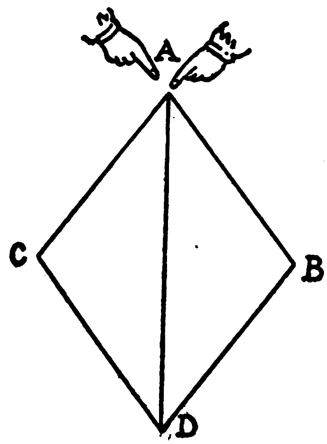

FIG. 17.

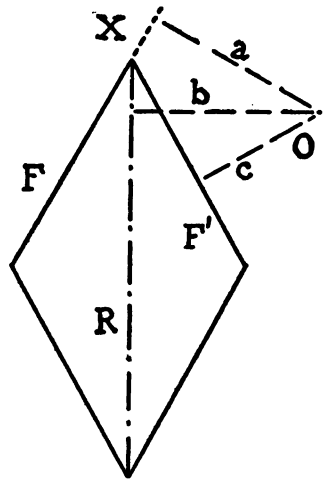

FIG. 18.

(Diagram from "Nouvelle Mecanique.")

(another line), the product of which is a certain area, is equal to the complementary moment also represented by an equal area in the diagram. For example, in the diagram, Fig. 12, from the Nouvelle Mecanique we have then $F \times a + F' \times c = R \times b$ or the moment of the diagonal of a parallelogram of forces is equal to the sum of the moments of the other two sides. The point O may be chosen either without the parallelogram within it or on one of the sides. Varignon demonstrates all three cases by proving that the areas are equal by geometry.

If the point O be taken within the parallelogram and the perpendiculars be then drawn we have $F \times a - F' \times c = R \times b$. Finally if O be taken on the diagonal the moment of the diagonal is zero and we have $Fa = F'c$. In every case the proof consists in a geometrical proof of equality of areas, the area being the representation of the product of a line, the lever arm, by another line representing the force. The principle thus proven is often called Varignon's principle. It is hardly credible that this principle of moments was not established until 1687, but such appears to be the fact. The proof of

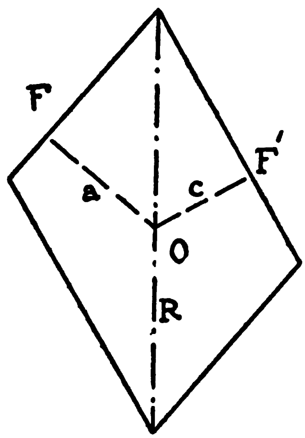

FIG. 19.

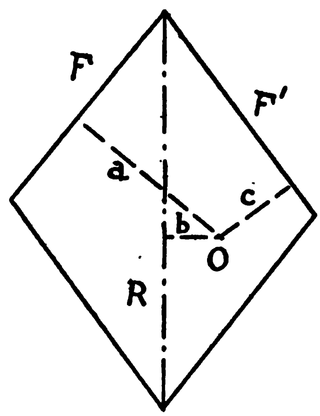

FIG. 20.

this principle was quite within the reach of Archimedes, but was not established until nearly twenty centuries after his time.

As an inevitable corollary of Varignon's proof of the principle of moments we arrive at the mechanical rule that in all cases of the parallelogram of forces and in all cases of statical equilibrium of forces in a plane the algebraic sum of the moments of the forces must be zero.

Having established the principle of moments, Varignon then applies it to many examples of equilibrium in rigid bodies and in machines. He used it for the solution of all problems of all the simple machines and founded a whole system of statics on this idea of balanced moments.

For the exposition of the simple machines it is perhaps easier for the student to grasp than the principle of virtual velocities which was established earlier.

Varignon in his Mecanique refers all cases of equilibrium back to his proof of moments as a criterion and presents therefore a harmonious theory of mechanics. His exposition is essentially geometrical and graphical, and is based upon Stevinus' triangle of forces the proof of which Varignon states as axiomatic in Lemma II of his book. His method of presentation and his proofs are a great advance over those of Archimedes and Stevinus.

Many of the modern text-book methods in statics are copied directly from this Mecanique and used verbatim to this day in class-rooms. The method of the parallelogram of velocities and of forces and the method of moments as applied in the simple machines are in daily use among engineers.

Algebra and geometry deal with fixed quantities, but with the development of dynamics, mathematics was called upon to investigate and express quantities whose value is continually changing. In the latter half of the seventeenth century, this need was met by the invention of that branch of mathematics, called the calculus. As has been noted Newton invented one method of studying the relative changes in dependent quantities by considering the ratio of change of their elements, which method of studying "flowing quantities" he called fluxions.

At about the same time Leibnitz, feeling the need of some such method, developed his system of studying change by infinitely small differences or by the "method of *infinitesimals*." The fundamental idea and the purpose of the two systems is much the same. Each calculus consists of two branches: (1) differential calculus which comprises methods of deducing the relations between infinitely small differences of quantities from the relations of the quantities themselves; (2) the integral calculus which treats of the inverse process of determining the relations of the quantities themselves when the relations of their infinitely small differences is known.

Newton's theory of flux or flow was better suited to Mechanics than Leibnitz's concept of instantaneous changes but the latter's notation was found more serviceable and has generally displaced Newton's symbols. It was perhaps a hundred years before this method was generally accepted,

6

recognized and developed. When we consider that all nature varies continually, the importance of this mathematical method of treating variables is obvious. With this instrument once mastered by the investigators, advance in Mechanics became rapid.

The transactions of the learned societies of this period are filled with discussions as to the integrity of the calculus methods, and with numerous isolated memoirs on its utility in one problem or another in mechanics. Among the earliest and most noteworthy of these is perhaps that on page 22, "Memoirs de l'Academie des Sciences de Paris, 1700," in which Varignon clearly presents for the first time, the differential equations of motion:

$$\frac{dx}{dt} = v, \qquad \frac{dv}{dt} = f,$$

$$\frac{d^2x}{dt^2} = f, \qquad v\frac{dv}{dx} = f.$$

These equations express completely the circumstances of rectilinear motion for every condition of acceleration or retardation. Newton had stated these laws geometrically (Principia, Lib. I, sect. 7; Lib. II, sect. 1) but Varignon seems to have been the first to advocate their expression in the notation of the calculus. He aspired to free Dynamics from the encumbrance of purely geometrical proofs by using this lately invented method of the calculus, and showed how acceleration might be expressed by the calculus, thus helping to clear the way for an analytical mechanics.

Like his predecessors he was greatly interested in hydrostatics and hydraulics and is to be credited with the earliest clear proof of the important principle that the velocity of efflux of a liquid is equal to $\sqrt{2gh}$.

Starting with the relation between force and the momentum $Ft = mv$ and denoting the area of the orifice by a, the head of liquid by h, specific gravity by S, acceleration due to gravity by g, the velocity of efflux by v, and by T a small interval of time we have,

$$ahS.T = \frac{avTS}{g} \cdot v,$$

$$gh = v^2.$$

In the formula ahS represents the pressure acting during the time T on the mass of liquid $avTS/g$. But since v is a final velocity we get more exactly,

$$ahS.T = \frac{a\frac{v}{2}TS}{g} \cdot v$$

or

$$v^2 = 2gh \quad \text{or,} \quad v = \sqrt{2gh}.$$

Varignon's contributions may be summed up then as:

1. Proof of the principle of moments.
2. A complete system of statics based on moments.
3. The differential equations $\begin{cases} \dfrac{dx}{dt} = v, \\ \dfrac{dv}{dt} = a, \\ \dfrac{d^2x}{dt^2} = a. \end{cases}$
4. The equation for velocity of efflux in hydraulics, $v^2 = 2gh$.

That these are masterly contributions to the science of mechanics is self-evident.

REFERENCES.

Nouvelle Mecanique, ouvrage posthume de M. Varignon, 2 vols., 1725.
Memoirs d'l Academie des Sciences de Paris.
Geschichte der Principien der Mechanik, Dühring.
Mach's Science of Mechanics.

Leibnitz, the Bernoullis and Euler.

Two hundred years ago the facilities for the spread of scientific progress and invention were meagre, and in general unless the studies of the philosophers developed something that bore directly upon some immediate practical problem of the time, no general notice was taken of their work. However after the principles of Statics and Dynamics which the researches of Stevinus, Galileo, Huygens, Newton and Varignon had developed, came to be understood among scholars, the custom of sending out challenge problems arose.

These competitions developed many small points which in the aggregate amounted to a very considerable contribution. Slowly a universal method of mechanical reasoning and notation came into use, which was understood in Florence and Paris as well as in Berlin, London and St. Petersburg. It consisted in the reduction of questions concerning force and motion to problems in pure geometry and calculus.

This method which began with the crude picture diagrams of Stevinus grew into the formal abstract geometrical mechanics of Newton's "Principia" and by the genius of Varignon and others was then expressed analytically. Once the analytical method was developed it became the custom for the Philosophical Societies of Paris, London, Berlin and St. Petersburg to offer prizes for solutions to various problems in mechanics, and there resulted a period of great activity in the application and extension of the fundamental contributions of Stevinus, Galileo, Huygens, Varignon and Newton.

Thus the calculus of Newton and Leibnitz came to be applied in a great variety of problems in mechanics, and ultimately, this method displaced entirely the geometrical method. During this period both methods were often used. Numerous mechanical problems were proven by both methods separately and much was achieved in a disjointed unconnected way.

Most active among those who took part in this development were Gottfried Wilhelm von Leibnitz (1646–1716), Leonhard Euler (1707–1783), and the Bernoullis—James (1654–1705), John (1667–1748) and Daniel (1700–1782).

Leibnitz, through his papers in the "Acta Eruditorum," which he founded, familiarized continental writers with his powerful method of analysis. Though Newton probably developed the calculus earlier by the method of fluxions, he had presented his Principia on the geometrical method and the English for a long time followed the geometrical method. It was not till 1817 that the differential calculus was introduced into the curriculum at Cambridge and came into general use in England. On the continent however, the method of Leibnitz came into use almost immediately and generally. Leibnitz and James Bernoulli, his brother John, the latter's sons Nicolas and Daniel, and their friend Euler were most active in this work of applying the Leibnitzian analysis to various problems in mechanics.

This was a period of development during which there was often acrimonious controversy. Two men sometimes arrived at similar or very similar results by different routes and then entered into a wordy conflict over their methods, both of which often proved to be correct. But great and lasting good came of all these discussions for they served to clear up and define the fundamental concepts of the science and develop methods and forms of proof which later masters correlated into formal treatises.

Leibnitz does not appear to have had a perfectly clear conception of mass. He speaks of a body as "corpus" and of a load or weight as "moles" and only once does "massa" occur. He makes the distinction however between "vis mortua" (pressure) and "vis viva" (moving force). His ideas of the measure of force also were not clear. He noticed that in machines in equilibrium the loads are inversely proportional to the velocities of displacement, so he measured the force by the product of the body ("corpus" or "moles"), into velocity. So far he was in accord with the notion of Newton and Descartes who regarded momentum as the measure of force, but Leibnitz held that such measure of force is only accidental and that the true measure of force is determined by the method of Galileo and Huygens, viz: by the mass times the velocity squared.

In 1686 in the "Acta Eruditorum," Leibnitz attacked Descartes' conception under the title "A short Demonstration of a Remarkable Error of Descartes and Others concerning the Natural Law by which they think the Creator always preserves the same Quantity of Motion; by which however, the science of mechanics is totally perverted." This dispute continued to agitate philosophers, until the constancy of Σmv and Σmv^2 was realized. Certainly Leibnitz was not perfectly clear upon this. His discussion, however, helped toward a solution of the difficulty.

James Bernoulli, a friend and admirer of Leibnitz, applied the calculus to various problems in mechanics with marked success. He was the first to work out a formula for the isochronous curve and for the catenary curve.[3]

Galileo had assumed that a uniform flexible string supported at its two extremities and acted upon by gravity would hang in a parabolic curve. James Bernoulli successfully applied analysis to this problem of the "chainette" as he called it, and to derive the formula for the catenary curve. Having solved it himself he proposed it as a challenge problem in 1691. The four mathematicians who solved it successfully were Leibnitz, Huygens and James and John Bernouilli. Their solutions appear in the Acta Eruditorum for 1691, pages 273–282, and also in the Philosophical Transactions of the Royal Society at London, 1697.

From the physical point of view it is easily seen that equilibrium exists when all the links of the chain have sunk as low as possible, so that no link can sink lower without raising part of the chain of equal mass higher, in consequence of the connections. This state of equilibrium exists when the center of gravity has sunk as low as it can sink. The mathematical problem then resolves itself into the problem of determining the curve that has the lowest center of gravity for a given length between A and B. The equation of such a curve Bernoulli determined with the aid of the calculus.

James Bernoulli also extended the method of analysis to the study of the curve of an elastic rod with a weight fixed at the end. This he put forth under the title of "Elastica." He also discussed the problem of the flexible sheet or impervious sail filled with a liquid which he presented under the titles "linetaria" and "volaria," and investigated the theory of cycloidal lines and various spiral and logarithmic curves. His writings

[3]"Opera," Tom. I, p. 449.

include an edition of Algebraic Geometry and the "Ars Conjectandi," in which he established the fundamental notions of the calculus of probabilities.

Johann Bernoulli, brother of James, was the most prominent and successful professor of mathematics of his time, holding the chair at Basel and Groningen. His lectures contain the earliest use of the term "integral" and show the first effort to construct an integral calculus as a set of general rules or a body of mathematics. Before this, investigators had treated each problem of integration by itself. He made himself a master of the calculus and applied it with marked success to many problems.

He was the author of the famous challenge problem of the "brachistochrone" which he propounded, 1697.

In abbreviated form it is as follows:

"Acutissimis qui toto orbe florent Mathematicis"

Johannes Bernoulli, Math. P. P.

....

"Problema Mechanico-Geometricum de linea celerrimi descensus

"Determinare lineam curvam data duo puncta, in diversis ab horizonte distantiis, et non in eadem recta verticali posita, connectentem, super qua mobile, propria gravitate decurrens et a superiori puncto moveri incipiens, citissime descendat ad punctum inferius

....

Groningæ ipsis Cal. Jan. 1697."

Which may be freely translated:

A Challenge to the Keenest Mathematicians of the World, From John Bernoulli, Prof. of Mathematics.

A Mechanico-Geometrical Problem of the Curve of Swiftest Descent.

Find the curve of quickest descent between any two given points at different distances from a horizontal line and not in the same vertical straight line.

Groningen, January 1, 1697.

The correct solutions were given by Leibnitz (Acta Erudit., 1697, p. 203), by Newton (Phil. Trans., 1697, No. 224, p. 389), by L'Hopital (Acta Erudit., 1697, p. 217) and John Bernoulli (Acta Erudit., 1697, p. 207).

Bernoulli's methods are the methods of the present time as the following quotations from his "Lectiones Mathematice, 38, 39, 40, Opera, Tom. III," will show:

A flexible string fixed at any two points A and B is acted upon by gravity. If we suppose the mass of the string to vary according to any assigned law as we pass from one point to another, to find the equation of the catenary of rest. Conversely the curve being known, to determine the law of mass of the string.

Let the axis of y extend vertically upwards, and the axis of x be horizontal, the plane xOy coinciding with the plane which contains the catenary. Then since,

$$x = 0, \quad y = -g,$$

we have, by previous equations, section I,

$$\frac{d}{ds}\left(t\frac{dx}{ds}\right) = 0, \qquad (a)$$

$$\frac{d}{ds}\left(t\frac{dy}{ds}\right) = mg. \qquad (b)$$

Integrating the equation (a) we get

$$t\frac{dx}{ds} = C.$$

Where C is a constant quantity.

Let T denote the tension at the lowest point, then evidently $T = C$, and therefore

$$t\frac{dx}{ds} = T. \qquad (c)$$

From (b) and (c), we have

$$T\frac{d}{ds}\frac{dy}{dx} = mg.$$

And therefore,

$$T\frac{dy}{dx} = \int mgds, \tag{d}$$

but at the lowest point of the catenary $dy/dx = 0$ and therefore, supposing a to be the value of S, at the lowest point,

$$T\frac{dy}{dx} = g\int_a^s mds. \tag{e}$$

If m be given in terms of the variable x, y, s the form of the catenary may be determined from (d).

Again, differentiating (d) we obtain

$$m = \frac{T\dfrac{d^2y}{dx^2}}{g\dfrac{ds}{dx}},$$

a formula by which m may be computed for every point of the string when the form of the catenary is given. Also from (c) we get

$$t = T\frac{ds}{dx}, \tag{f}$$

which gives the tension at any point of the catenary when its form is known.

Another example is that on page 497, Tom. III, Lectiones Mathematice; Opera.

A flexible string AOB fixed at two points A and B is acted upon by gravity, the mass at any P varies inversely as the square root of the length OP measured from the lowest point O; to find the equation of the catenary.

Let the origin of co-ordinates be taken at O, x being horizontal, and y vertical, and the plane of xy coinciding with the plane of the catenary, also let O be the origin of S.

Then, if μ be the mass at end of a length C from the lowest point,

$$m = \mu\frac{C^{\frac{1}{2}}}{S^{\frac{1}{2}}},$$

and therefore 1, d, α being in the present case zero, we have

$$T\frac{dy}{dx} = g\mu c^{\frac{1}{2}}\int_0^s \frac{ds}{S^{\frac{1}{2}}} = 2g\mu c^{\frac{1}{2}}s^{\frac{1}{2}},$$

hence putting for sake of brevity

$$\frac{2g\mu c^{\frac{1}{2}}}{T} = \frac{1}{\beta^{\frac{1}{2}}},$$

we get

$$\frac{dy}{dx} = \left(\frac{S}{\beta}\right)^{\frac{1}{2}}\frac{dy^2}{dx^2} = \frac{S}{\beta},$$

$$\beta\frac{d}{dx}\frac{dy^2}{dx^2} = \frac{ds}{dx} = \left(1 + \frac{dy^2}{dx^2}\right)^{\frac{1}{2}},$$

$$\frac{\beta\dfrac{d}{dx}\dfrac{dy^2}{dx^2}}{\left(1 + \dfrac{dy^2}{dx^2}\right)^{\frac{1}{2}}} = 1;$$

integrating with respect to x we obtain,

$$2\beta\left(1 + \frac{dy^2}{dx}\right)^{\frac{1}{2}} = x + C;$$

but $x = 0$, $dy/dx = 0$ simultaneously; hence $C = 2B$ and therefore

$$2\beta\left(1 + \frac{dy^2}{dx^2}\right)^{\frac{1}{2}} = x + 2\beta; \qquad (a)$$

squaring and transposing

$$4\beta^2\frac{dy^2}{dx^2} = \left(x + 2\beta\right)^2 - 4\beta^2,$$

$$2\beta dy = \{(x + 2\beta)^2 - 4\beta^2\}^{\frac{1}{2}}dx;$$

integrating we have

$$C + 2\beta y = \tfrac{1}{2}(x + 2\beta)(x^2 + 4\beta x)^{\frac{1}{2}} - 2\beta^2\log\{x + 2\beta + (x^2 + 4\beta x)^{\frac{1}{2}}\}.$$

But $x = 0$, $y = 0$, simultaneously; hence

$$C = 2\beta^2\log(2\beta),$$

hence, eliminating C,

$$2\beta y = \tfrac{1}{2}(x + 2\beta)(x^2 + 4\beta x)^{\frac{1}{2}} - 2\beta^2 \log \frac{x + 2\beta + (x^2 + 4\beta x)^{\frac{1}{2}}}{2\beta},$$

which is the required equation of the catenary.

Cor. From (a) we get

$$\frac{ds}{dx} = \frac{x + 2\beta}{2\beta},$$

and therefore, by (1, f)

$$t = T\frac{ds}{dx} = \frac{T}{2\beta}(x + 2\beta),$$

which gives the tension at any point of the curve.

On page 502, of Tom. III, Opera, we find the interesting problem:

To find the law of variation of the mass of a catenary acted upon by gravity so that it may hang in the form of a semi-circle with its diameter horizontal.

The notation remains the same as in the preceding problem, and the equation of the catenary is

$$x^2 = 2ay - y^2,$$

where a denotes the radius of the semi-circle; hence

$$a^2 - x^2 = (a - y)^2,$$

$$y = a - (a^2 - x^2)^{\frac{1}{2}},$$

$$\frac{dy}{dx} = \frac{x}{(a^2 - x^2)^{\frac{1}{2}}}, \quad \frac{d^2y}{dx^2} = \frac{a^2}{(a^2 - x^2)^{\frac{3}{2}}};$$

also

$$\frac{ds^2}{dx^2} = 1 + \frac{dy^2}{dx^2} = \frac{a^2}{a^2 - x^2}, \quad \frac{ds}{dx} = \frac{a}{(a^2 - x^2)^{\frac{1}{2}}},$$

and therefore by (1, e)

$$m = \frac{T\dfrac{d^2y}{dx^2}}{g\dfrac{ds}{dx}} = \frac{Ta}{ga^2 - x^2} = \frac{Ta}{g(a - y)^2},$$

or the mass at any point varies inversely as the square of the depth below the horizontal diameter of the circle. Cor. By (1, *f*) we have for tension at any point

$$t = T\frac{ds}{dx} = \frac{Ta}{(a^2 - x^2)^{\frac{1}{2}}} - \frac{Ta}{a - y^2}$$

These proofs show great facility in handling the calculus but they are an extension of known ideas rather than a new contribution. Nevertheless the methods of Johann Bernoulli exerted a great influence upon the development of the science in extending the mathematical or analytical method of treatment.

As to his new contributions, he set forth the principle of virtual velocities in a letter to Varignon, introduced the symbol g, and assisted him in arriving at the formula $v^2 = 2gh$, which had been previously stated,

$$v_1^2 : v_2^2 :: h_1 : h_2.$$

This Bernoulli was a profound scholar and wrote on a great variety of topics as will be seen from the following selection of titles from his Opera Omnia published at Lausanne in 1742.

I. Dissertatio de Effervescentia and Fermentatione.
II. Novum Theorema pro Doctrina Conicarum.
III. Inventio curvæ geometricæ quæ spirali æquatione.
IV. Solutio Problematis Funicularii.
V. Curvæ sui evolutione se ipsas describentes.
XVIII. Dissertatio physico anatomica de motu musculorum.
LIII. Disputatio medico physica de nutritione.
XC. De motu pendulorum et projectilium.
XCIX. Demonstratio principii Hydraulici de velocitate per foramen et vase erumpentis.
CXL. Meditationes de Chordis vibrantibus.
XXIV. Cycloidis evoluta ipsa est cyclois.
XXXIII. Varia Problemata Physico-Mechanica.

The most interesting of these is the first part of the third volume, "Discours sur les Loix de la communication du mouvement, contenant la solution de la premiere Question proposée par Messieurs de l'académie Royale des Sciences pour l'annee 1724."

As a preface to it Bernoulli says: "The author of this discourse has the honor to present it to the Academy. It was composed on the occasion of the first of the questions propounded by the Society to the savants of Europe. Messrs. Huygens, Mariotte, Wren, Wallis and various other mathematicians have written worthily on this subject and given us rules for impulse. But not satisfied with taking a general rule for the most simple cases, by a kind of induction, the author has followed a method different from theirs and more natural.

"He goes back to the sources and taking up all that is known of the subject, it is on principles of mechanics that he deduces like corollaries particular rules for each case. Up to this time we have had only a confused idea of the force of bodies in motion to which M. Liebnitz has given the name vis viva. The author has not only attempted to bring the discussion down to date and to explain the difficulty between Leibnitz and those of the opposite party, he has attempted to prove by demonstrations direct and entirely new, a truth which M. Leibnitz never proved except indirectly.

"He proposes to show that the vis viva of a body is not proportional to its simple velocity as commonly believed but to the square of the velocity and he hopes to prove what he shall say, so that no one shall any longer doubt the truth of this proposition: moreover, he proposes to determine that which results from the shock of a body which encounters two or several others following different directions, a problem so difficult that no one has yet solved it. And indeed, how could any one, since its solution presupposes an exact comprehension of the theory of vis viva?

"This theory opens an easy way to several important truths. It has given the author a solution of the preceding problem which seems somewhat peculiar and a method of determining the actual loss of velocity in a resisting medium and an easy way of finding the center of oscillation in compound pendulums." He then goes on to expound the principle of virtual velocities and the notion of energy as measured by the mass and the velocity squared.

Talent for mathematics seems to have run in the Bernoulli family. Several of the younger generations were famed for their writings and teaching, among them, the three sons of John, viz: Nicholas, Daniel and John, Jr., and the two sons of John, Jr., John, 3rd, and Jacob.

Of these Daniel Bernoulli (1700–1782) was the most prominent. He was professor at St. Petersburg and at Basel, a famous mathematician, and winner of the French Academy prize ten times. His chief work in mechanics was on hydrodynamics and the solution of the problems of vibrating cords, in which we find ingenious extensions of known principles of mechanics by the aid of the calculus.

Euler (1707–1783), the pupil and friend of Johann Bernoulli and friend of his sons, carried the integral calculus to a high degree of perfection and invented numerous solutions of mechanical problems. His strength lay rather in pure than in applied mathematics. Euler's principal contributions are set forth in his "Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes" (1744) in which he presents the method of co-ordinate analysis and shows the properties of maximum and minimum of various curves.

He also published at St. Petersburg in 1736 his "Mechanica sive Motus Scientia Analytice Exposita" which is sometimes referred to as the first book of Analytical Mechanics. In this, he still adheres in part to the old method of geometrical presentation of mechanics, but his general method is to resolve all forces in three fixed directions, X, Y, Z. This makes his presentations and computations lucid and symmetrical.

As an example, note the method of the following discussion from page 237, Tom. I of "Mechanica," on tangential and normal resolution in curvilinear motion.

A particle is projected with a given velocity in a given direction, and is acted upon by a constant force in parallel lines, to determine the path of the particle.

Let the axis of X be taken so as to pass through the initial place of the particle, and let the axis of Y be taken parallel to the constant force which acts toward the axis of X. Let f denote the constant force.

Then, the tangential resolved part being $-f\frac{dy}{ds}$, and the normal one being $f\frac{dx}{ds}$ we have for the motion of the particle

$$v\frac{dv}{ds} = -f\frac{dy}{ds}, \qquad (1)$$

$$\frac{v^2}{\rho} = f\frac{dx}{ds}. \qquad (2)$$

Integrating (1)

$$v^2 = C - 2fy.$$

Let V be the initial velocity; then y being zero initially $V^2 = C$; therefore

$$v^2 = V^2 - 2fy.$$

Hence substituting this expression for v^2 in (2)

$$\frac{1}{\rho} = (V^2 - 2fy) = f\frac{dx}{ds},$$

but

$$\rho = -\frac{\frac{ds^2}{dx^2}}{\frac{d^2y}{dx^2}};$$

hence

$$-\frac{d^2y}{dx^2}(V^2 - 2fy) = f\frac{ds^2}{dx^2} = f\left(1 + \frac{dy^2}{dx^2}\right),$$

$$(V^2 - 2fy)\frac{d}{dx}\left(1 + \frac{dy^2}{dx^2}\right) - \left(1 + \frac{dy^2}{dx^2}\right)\frac{d}{dx}(V^2 - 2fy) = 0.$$

Integrating, we have

$$C\left(1 + \frac{dy^2}{dx^2}\right) = V^2 - 2fy.$$

Where C is an arbitrary constant.

Let β be the angle which the direction of projection makes with the axis of x; then

$$C(1 + \tan^2 \beta) = V^2,$$

hence

$$V^2\left(1 + \frac{dy^2}{dx^2}\right) = \sec^2 \beta \, (V^2 - 2fy),$$

$$V^2 \frac{dy^2}{dx^2} = V^2 \tan \beta - 2fy \sec^2 \beta,$$

$$Vdy = (V^2 \tan^2 \beta - 2fy \sec^2 \beta)^{\frac{1}{2}} \, dx;$$

whence by integration,

$$C - V(V^2 \tan^2 \beta - 2fy \sec^2 \beta)^{\frac{1}{2}} = fx \sec^2 \beta.$$

But $x = 0$, $y = 0$ simultaneously; hence

$$C - V^2 \tan \beta = 0,$$

and therefore

$$V^2 \tan \beta - V(V^2 \tan^2 \beta - 2fy \sec^2 \beta)^{\frac{1}{2}} = fx \sec^2 \beta.$$

Clearing the equation of radicals and simplifying, we obtain,

$$y = \tan \beta . x - \frac{f \sec^2 \beta}{2V^2} x^2.$$

He also solves in a similar manner various problems such as:

"A particle always acted on by a force in parallel lines, describes a given curve; to determine the nature of the force, the velocity and the direction of projection being given."

And, "A particle describes a given curve about a center of force; to determine the motion of the particle and the law of force."

As has been stated the name Moment of Inertia of a body was given by Euler to the sum of all the products resulting from the multiplication of each element of the mass by the square of the distance from the axis. In his "Theoria Motus Corporum Solidorum," page 167, Euler says: "Ratio hujus denominationis ex similitudine motus progressivi est desumpta: quemadmodum enim in motu progressivo, si a vi secundum suam directionem sollicitante acceleretur, est incrementum celeritatis ut vis sollicitans divisa per massam seu inertiam; ita in motu gyratorio, quoniam loco ipsius vis sollicitantis ejus momentum considerari oportet, eam expressionem $\int r^2 dM$ quæ loco inertiæ in calculum ingreditur, *momentum inertiæ* appelemus, ut incrementum celeritatis angularis simili modo proportionale fiat momento vis sollicitattis diviso per momentum inertiæ."[1] This very useful expression used so commonly by engineers was developed by Euler for various plane figures and for solids of revolution. His method for finding the moment of inertia for the sphere, right cone, cylinder and other figures is given on page 198 and the following pages of "Theoria Motus Corporum Solidorum," as follows:

To find the radius of gyration of a homogeneous sphere about a diameter. Let x, $x + dx$ be the distances of the circular faces of a thin circular slice of a sphere at right angles to the diameter, from the center and let y be the radius of the section; then ρ denoting the density of the sphere, the moment of inertia of this slice about the diameter will be equal to

$$\tfrac{1}{2}\pi\rho y^4 dx,$$

and therefore the moment of inertia of the whole sphere, a being its radius, will be equal to

$$\tfrac{1}{2}\pi\rho \int_{-a}^{+a} y^4 dx = \tfrac{1}{2}\pi\rho \int_{-a}^{+a} (a^2 - x^2)^2 dx = \tfrac{8}{15}\,\pi\rho a^5.$$

As the mass of the sphere is $\tfrac{4}{3}\pi\rho a^3$ the radius of gyration

$$k^2 = \tfrac{2}{5}a^2.$$

Similarly on page 203 the radius of gyration of a hollow

[1] *Translation.*—The scheme of this notation is derived by analogy with rectilinear motion; for as in rectilinear motion if it be increased by a disturbing force in its own direction the increase of velocity (acceleration) is equal to the disturbing force divided by the mass or inertia, thus in rotary motion, since in place of the disturbing force itself we must consider its moment we call that expression $\int r^2 dM$ which comes into calculation in place of inertia—the moment of inertia—so that the increase of angular velocity in a similar way is made proportional to the moment of the disturbing force, divided by the moment of inertia.

sphere with external and internal diameters *a*, *b*, is proven to be

$$k^2 = \frac{2}{5}\frac{a^5 - b^5}{a^3 - b^3}.$$

Euler systematized and perfected the mathematical knowledge of the time. Among his publications are,

Introductio in analysin infinctorum..........1748
Institutiones Calculi differential..............1755
Institutiones Calculi Integral................1768

also a development of the Calculus of Variations.

He set forth the principle of least action,—though Maupertuis is usually given the credit of having originated the notion,—expressing it in that curious blending of theology and science common in this period, in this fashion: the all-wise Maker would not make anything in which some maximal and minimal property is not shown.

The original Latin form is "Quum enim universi fabrica sit perfectissima, ataque a creatore sapientissimo absoluta, nihil omnino is mundo contingit in quo non maximi minimive ratio quæpiam eluceat; quam ob rem dubium prorsus est nullum, quin omnes mundi effectus ex causis finalibus ope methodi maximorum et minimorum, æque feliciter determinari quæant, atque ex ipsis causis efficientibus," or "For since the fabric of the universe is most perfect and finished by a most wise creator, nothing occurs in the world in which some plan of maxima and minima does not show forth; *therefore there is no doubt at all* (*!*) but that all phenomena of the world are equally well to be determined from final causes by the method of maxima and minima and from the same effecting causes."

This idea was taken up by Euler (Proc. Berlin Acad., 1751) and developed into a theory of equilibrium of utility by the application of the method of maxima and minima. If in any system we cause infinitely small displacements we produce a sum of virtual moments

$$Pp + P'p' + P''p'' + \cdots$$

which only reduces to zero in the case of equilibrium. The sum is the work corresponding to the displacements, or since for minute displacements it is itself infinitely small, the corresponding element of work. If the displacements are continuously increased till a finite displacement results, their summation is a finite amount of work.

Therefore if we start with any initial configuration of the system and pass to any given final configuration, a certain amount of work will have to be done. This work done when a final configuration or a configuration of equilibrium or equilibrium is reached is a maximum or a minimum. That is if any system is carried through the configuration of equilibrium the work done is previously and subsequently less or greater than at the configuration of equilibrium itself. For equilibrium, therefore

$$Pp' + P'p' + P''p'' + \cdots = 0.$$

From this Euler deduced the principle that the element of work or the differential of work is equal to zero in equilibrium; and if the differential of a function can be put equal to zero, the function has generally a maximum or minimum value.

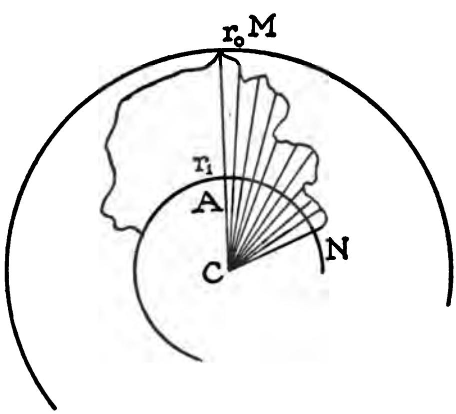

FIG. 21.

This highly ingenious method of determining the equilibrium of a system was later developed by others. In 1749 Courtivron, in a paper before the Paris Academy gave it the form: "For the configuration of stable or unstable equilibrium at which work done is a maximum or a minimum, the vis viva of the system in motion, is also a maximum or minimum in its transit through these configurations."

Euler also assisted in the development of the so-called principle of vis viva. He showed that if a body M is attracted to a fixed center C according to a certain law the increase in vis viva in the case of rectilinear approach is calculable from the initial and terminal distances r_0, r_1. But the increase is the same if M passes at all from the position r_0 to r_1 independently of the form of the path MN. The elements of the work done are to be calculated from the projections on the radius of the actual displacements and are thus ultimately the same.

Euler is also to be credited with the first general use of π for 3.1416 + and the application of his methods of analysis to hydrodynamics.

We may sum up his contributions then as follows:

1. A perfect systematizing of the calculus.
2. The foundations of analytical mechanics.
3. The analytical method of resolving tangential and normal components of curvilinear forces.
4. The development of moment of inertia.
5. The principle of least action or maxima and minima in equilibrium.
6. The principle that the increase of vis viva is independent of the path.

Much of the work of D'Alembert and Lagrange is based on the contributions or methods of Euler, and perhaps would not have been possible without Euler's work.

REFERENCES.

Jacobi Bernoulli Basilensis Opera. Geneva, 1744.
Johanni Bernoulli Basilensis Opera Omnia. 1742.
Opuscules et Fragments inedits de Leibnitz. Paris, 1903.
Euler, Mechanica Sive Motus, 1736.
Euler, Institutiones Calculi Differentialis.
Euler, Introductio in Analysin Infinitorum.
Correspondenz von Nicolaus Bernoulli. Basel Library.
Nouvelle Mecanique. Varignon. Paris, 1725.
Euler, Methodus, 1744.
Harnack, Leibnitz's Bedeutung in der Geschichte der Mathematik.
D. Bernoulli, Hydrodynamica.
Cantor, Geschichte der Mathematik.

JEAN-LE-ROND D'ALEMBERT (1717–1783).

As a result of the labors of a host of contributors much had now been evolved in mechanics in a disjointed way and from diverse points of view. The prize and challenge problems were usually very special and did not tend to develop a formal presentation of the science. It was now in order for some one to verify, consolidate and formulate all these contributions.

This D'Alembert did in his "Traite de Dynamique" (1743.) While his work rests upon the work of all his predecessors, and while he is particularly indebted to Euler, yet his Treatise possesses distinctly original features. He shows that all problems in dynamics may be regarded as problems in statics and he applies in their solution one single unifying principle known by his name as D'Alembert's principle. It is to the effect that in any system of bodies the impressed forces are equivalent to the effective force.

This formal presentation of mechanics in a treatise is a memorable event. It typifies the coming of age of the science. Henceforth it has a character and unity which it did not previously possess. This is due to the fact that now there is one general guiding principle, D'Alembert's Principle, to which all problems in mechanics can be referred for solution. Namely:—

If a material system connected together in any way, and subject to any constraints, be in motion under the influence of any forces, each point of the system has at any instant a certain acceleration. If now to each point an acceleration were imparted equal and opposite to its actual acceleration, the velocities of all points of the system would become constant, that is, each particle would move as if free and unacted on by any force whatever. The applied accelerations, the external forces, and the constraints and mutual or internal forces of the system, would equilibrate one another.

In the "Traite de Dynamique" this idea of which the above is a condensed translation is expressed as follows:—

"Problème General."

"Soit donne un systeme de corps disposés les uns par rapport aux autres d'une maniere quelconque; et supposons qu'on imprime à chacun de ces corps un mouvement particulier, qu'il ne puisse suivre à cause de l'action des autres corps; trouver le mouvement que chaque corps doit prendre."

"Solution."

"Soient A, B, C, etc., les corps qui composent le systeme et supposons qu'on leur ait imprime les mouvemens a, b, c, etc., qu'ils soient forcés, à cause de leur action mutuelle, de changer dans les mouvements a, b, c, etc. Il est clair qu'on peut regarder les mouvemens b, c, etc., comme composes des mouvemens $b, \beta; c, \gamma$; etc.; d'ou il s'ensuit que le mouvement des corps A, B, C, etc.; entr' eux auroit ete le même, si au lieu de leur donner les impulsions a, b, c, etc., on leur eut donné a-la-fois les doubles impulsions a, α; b, β; C, γ, etc. Or par la supposition, les corps A, B, C, etc., out pris d'eux-memes les mouvemens a, b, c, etc., donc les mouvemens α, β, γ, etc., doivent etre tels qu'ils ne derangent rien dans les mouvemens a, b, c, etc., c'est a-dire que, si les corps n'avoient recu que les mouvemens, α, β, γ, etc., ces mouvemens auroient dû se detruire mutuellement et le système demeurer en repos.

"De la resulte le principe suivant, pour trouver le mouvement de plusieurs corps qui agissent les uns sur les autres. De Composez les mouvemens a, b, c, etc., imprimés à chaque corps, chacun en deux autres, a, α; b, β; c, γ; etc.; qui soient tels, que si l'on n'eût imprimé aux corps que les mouvemens, a, b, c, etc., ils eussent pu conserver ces mouvemens sans se nuire reciproquement; et que si on ne leur eût imprimé que les mouvemens α, β, γ, etc., le systeme fut demeuré en repos; il est clair que a, b, c, etc., seront les mouvemens que ces corps prendront en vertu de leur action. Ce qu'il falloit trouver."

The idea was not entirely new. James Bernoulli in a memoir published in Acta Eruditorum, 1686, p. 356, "Narratio Controversiæ inter Dn. Hugenuim et Abbatem Catelanum agitatæ de Centro oscillationis," set forth the idea of reducing the determination of the motions of material systems to the solution of statical problems. It is a direct consequence of Newton's laws rather than a new principle. However, to D'Alembert belongs the credit of clearly setting forth this idea and of founding a formal mechanics upon it.

In algebraic language the principle is: If the co-ordinates of any particle m of a material system be x, y, z and the external forces there applied be X, Y, Z the system of forces

$$X_1 - m_1\frac{d^2x}{dt^2}, \quad Y_1 - m_1\frac{d^2y}{dt^2}, \quad Z_1 - m_1\frac{d^2z}{dt^2},$$

$$X_2 - m_2\frac{d^2x}{dt^2}, \quad Y_2 - m_2\frac{d^2y}{dt^2}, \quad Z_2 - m_2\frac{d^2x^2}{dt^2},$$

etc., acting at the points x, y, z and x_2, y_2, z_2, etc., will be in equilibrium in virtue of the constraints and mutual reactions of the system.

The force whose components are

$$-m\frac{d^2x}{dt^2}, \quad -m\frac{d^2y}{dt^2}, \quad -m\frac{d^2z}{dt^2},$$

is called the force of inertia of the mass m. D'Alembert's principle states that. The applied forces and the forces of inertia in any system are in equilibrium.

If in any problem the work be 0, the particular case of the principle of virtual displacement results. This principle follows therefore as a special case of D'Alembert's principle.

The equation of vis viva also follows from D'Alembert's principle. The integral of the equations of motion can usually be obtained from D'Alembert's principle, viz:

$$\Sigma\left\{\left(X - m\frac{d^2x}{dt^2}\right)\delta x + \left(Y - m\frac{d^2y}{dt^2}\right)\delta y + \left(Z - m\frac{d^2z}{dt^2}\right)\delta z\right\} = 0.$$

Here δx, δy, δz are arbitrary displacements consistent with the conditions of the problem. When the equations of condition do not contain the time explicitly, dx (the actual movement along the axis of x during an infinitely short time) is always a value which can be assigned to δx. In most problems dx is a possible value of δx and the same holds for dy and dz similarly. Therefore if this be admitted as a legitimate substitution as is usually the case, if we write dx, dy, dz for δx, δy, δz, D'Alembert's equation becomes

$$\Sigma m\left(\frac{d^2x}{dt^2}dx+\frac{d^2y}{dt^2}dy+\frac{d^2z}{dt^2}dz\right)=\Sigma(Xdx+Ydy+Zdz).$$

Integrating we have

$$\Sigma m\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right\}=2\Sigma\int(Xdx+Ydy+Zdz)+C.$$

This is the equation of vis viva.

If the vis viva at any particular t' is Σmv^2 we have

$$\Sigma mv^2-\Sigma mv'^2=2\Sigma\int(Xdx+Ydy+Zdz).$$

If there be no forces acting on the system its vis viva remains constant. The equations of vis viva are among the most important in dynamics. They are the foundation of the theory of energy.

By means of D'Alembert's principle the equation of motion of a rigid body can be written at once. We have only to write the six equations of equilibrium, taking into account applied forces and the forces of inertia and we have at once

$$\Sigma m\frac{d^2x}{dt^2}=\Sigma X,\quad \Sigma m\frac{d^2y}{dt^2}=\Sigma Y,\quad \Sigma m\frac{d^2z}{dt^2}=\Sigma Z,$$

$$\Sigma m\left(y\frac{d^2z}{dt^2}-z\frac{d^2y}{dt^2}\right)=\Sigma(yZ-zY),$$

$$\Sigma m\left(z\frac{d^2x}{dt^2}-x\frac{d^2z}{dt^2}\right)=\Sigma(zX-xZ),$$

$$\Sigma m\left(x\frac{d^2y}{dt^2}-y\frac{d^2x}{dt^2}\right)=\Sigma(xY-yX).$$

These equations express the moments about the axes.

The comprehensive character and broad application of D'Alembert's principle are apparent; other principles follow from it as corollaries. It supplies a routine-form of solution for problems, in a masterly fashion, with great economy of thought.

In his study of equilibrium and motion in fluids, and in the theory of vibrating strings D'Alembert encountered a partial differential equation of the forms,

$$\frac{\delta^2 u}{\delta t^2} = \frac{\delta^2 u}{\delta x^2},$$

which he finally solved in 1747. The solution is given in a paper before the Berlin Academy as follows:

If $\frac{\delta u}{\delta x}$ be denoted by p, and $\frac{\delta u}{\delta t}$ by q, then $du = pdx + pdt$. But

$$\frac{\delta q}{\delta t} = \frac{\delta p}{\delta x}$$

by the given equation, therefore $pdt + qdx$ is also an exact differential, denote it by dv.

Therefore

$$dv = pdt + qdx.$$

Hence

$$du + dv = (pdx + qdt) + (pdt + qdx) = (p + q)(dx + dt)$$

and

$$du - dv = (pdx + qdt) - (pdt + qdx) = (p - q)(dx - dt).$$

Thus $u + v$ must be a function of $x + t$ and $u - v$ must be a function of $x - t$. We may therefore put

$$u + v = 2\phi(x + t),$$

$$u - v = 2\psi(x - t).$$

Hence

$$u = \phi(x + t) + \psi(x - t)$$

in which ϕ and ψ are arbitrary functions.

In 1749 D'Alembert published the first analytical solution of the precession of the Equinoxes and of the rotation of the earth's axis. He also published a work entitled "Reflexions sur la Cause Generale des Vents," 1744, and three volumes on the "Systeme du Monde" in which his calculations and theories in astronomy are set forth.

One of D'Alembert's chief claims to distinction, in addition to his special contributions, is that he put Newton's results into the form of the Calculus and made possible their study and extension. He presented in his "Traité de Dynamique," the first treatise on Analytical Mechanics. When this had been done the way was prepared for a complete exhaustive treatment of the entire domain of Mechanics by the Analytical method. This was done by Lagrange within five years of D'Alembert's death.

REFERENCES.

Traité de Dynamique, 1743.
Traité de l'Equibré du Mouvement de Fluides, 1744.
Reflexions sur la Cause Generale des Vents, 1747.
Recherches sur la Precession des Equinoxes, 1749.
Reserches sur different Points Importans du Systeme du Monde, 1756.
Systeme du Monde, 3 vols., 1754.
W. W. R. Ball, History of Mathematics.
Mach, The Science of Mechanics.
Williamson, Treatise on Dynamics.
Bertrand, D'Alembert.
Condorcet, Eloge (French Acad., 1784).

4. The Contribution of Lagrange and Laplace.

Although it is probable that Newton used his method of fluxions or calculus in arriving at the ideas set forth in the Principia, still he presented them in geometrical form. Even so, it was some fifty years before they were accepted and assimilated. The next big step in advance was to be the full and complete development of mechanics by the analytical method based on Newton's laws.

It was necessary first that the calculus and its notation should be perfected and that its use and value in problems of mechanics should come to be recognized. The labors of Leibnitz, the Bernoullis and Euler brought this to pass. Secondly it was necessary that the co-ordinate method should be developed. This was done by Descartes, Euler and Maclaurin and D'Alembert. When this had been done it was possible to express the results of Newton in the language of the calculus and have them generally received and accepted.

JOSEPH LOUIS LAGRANGE (1736–1813).

Comte Lagrange, one of the greatest masters of pure and mixed mathematics that ever lived, was born at Turin though of French extraction. A Senator of France, a Count with the Grand Cross of the Legion of Honor, professor in the Artillery School of Turin and in the Polytechnic School of France, Director of the Berlin Academy under Frederick the Great, his life is one glorious record of achievement.

His great work the "Mécanique Analytique" is analytical as opposed to geometrical. There is not a geometric diagram in it, whereas the Principia is full of them, page on page. Written 100 years after Newton's great work, it is a grand comprehensive treatise gathering up the scattered methods and principles of the preceding century, harmonizing them and setting them forth in concise harmonious algebraic form. He gives a general method by which every mechanical question of *solids*, *liquids* or *gases* may be stated in a single algebraic equation. The entire mechanics of any system, even the solar system, can be summed up in a few equations by this method. This is a wonderful labor-saving and thought-saving device.

It was his boast that he had transformed Mechanics, (defined by him as "a geometry of four dimensions") into a branch of analysis. He exhibited the mechanical principles of his predecessors as simple results of the calculus, and introduced the method of regarding a fluid as a material system characterized by free mobility of its molecules. With this the separation between the mechanics of solids, liquids and gases disappeared, for the fundamental equations of forces could now be extended to hydraulics and pneumatics. He formulated a universal science of matter and motion, deduced from the principle of virtual velocities by the method of generalized co-ordinates.

Departing from the method of D'Alembert and Euler, instead of considering the motion of each individual part of a material system, Lagrange shows how to determine its configuration by a number of variables corresponding to the degrees of freedom of the system. The kinetic and potential energies of the system can be expressed in terms of these variables and the equations of motion obtained by differentiation.

He gave to analytical mechanics a complete logical perfection, reducing the science to differential equations and developing the calculus of variations. The introduction of the "Mécanique Analytique" (1788) is so simple and direct a statement of the author's purpose that it is worthy of literal quotation.

"There are already several treatises on Mechanics but the plan of this one is entirely new. I have attempted to reduce the theory of this science and the art of solving the problems connected with it to general formulas, whose simple development will have all the necessary equations for the solutions of each problem. I hope that the manner in which I have tried to accomplish my object will leave nothing to be desired.

"This work will have in addition another advantage: it will collect and present under the same point of view the different principles, so far found, to facilitate the solution of mechanical questions. It will show their connection, their mutual dependence, leaving one to judge of their accuracy and value."

"*No diagram will be found in the work.* The method which I follow requires neither figures nor arguments geometrical or mechanical, but merely algebraic operations arranged in a regular and uniform order. Those who are fond of analysis will anticipate this mechanics with pleasure, and be pleased that I have set it forth in this way."

Concerning the fundamental principle of the work, he says after stating D'Alembert's principle:

"But there is another manner of treatment more general and more severe which merits the attention of geometers. M. Euler gave the first hint of it at the end of his treatise on isomerism printed at Lausanne, 1744, showing that in the paths described by central forces, the integral of velocity by the element of the curve always is a maximum or a minimum. This property M. Euler had not noticed except in the motion of isolated bodies.

Since that time I have considered the motion of bodies acting upon each other in any fashion whatsoever, and there has resulted this new general principle that the sum of the products of the masses by the integral of velocities multiplied by the elements of the spaces covered is constantly a maximum or a minimum. Such is the principle to which I give here, although improperly, the name of "least action," and which I consider not as a metaphysical principle, but as a simple and general result of the laws of mechanics. One may see in volume 2, "Memoirs of Jarin," the use I have made of it for solving several difficult problems of dynamics."

This principle combined with that of the conservation of energy, and developed according to the rules of the calculus of variations, gives directly all the necessary equations for the solution of any problem.

He then proceeds to develop his "general dynamic formula for the motion of a system of bodies acted upon by any forces whatsoever," after the manner briefly indicated here. He says if the forces acting upon a body do not mutually destroy or equilibrate themselves as in statics, then the forces produce accelerations. When these forces act freely and uniformly they necessarily produce velocities which increase with the time. One may regard these velocities as measures of the forces. Let us suppose now that of every accelerating force we know the velocity that it is capable of impressing upon a free body during a unit time. We measure accelerating force by the velocity it produces in a unit time supposing the body to move uniformly for that time, and we know by the theorems of Galileo that this space that the body would pass over is twice the distance that the body moves under a constant accelerating force, such as gravity; therefore we have as the velocity by which to measure a constant force twice the distance that the body passes over in a unit time. We must choose our units accordingly.

After a careful development of these notions Lagrange says let us now consider a system of bodies disposed as you will and acted upon by any accelerating forces you please. Let m be the mass of any of the bodies regarded as point and let it be referred to three co-ordinate axes by the co-ordinates x, y, z at any instant t, then dx/dt, dy/dt, dz/dt, will represent the velocities in the directions of the axes, if the body is abandoned to itself and moves uniformly. But if by reason of the action of accelerating forces the velocities take on, during the instant t, the increments

$$d\frac{dx}{dt}, \quad d\frac{dy}{dt}, \quad d\frac{dz}{dt},$$

one may regard these increments as new velocities and dividing them by dt, one will have a measure of the accelerating forces that produce them. Taking the element of time dt, as constant, the accelerating forces will be proportional to d^2x/dt^2, dy^2/dt^2, dz^2/dt^2, and multiplying these forces by the mass of the body upon which it acts we have

$$m\frac{d^2x}{dt^2}, \quad m\frac{d^2y}{dt^2}, \quad m\frac{d^2z}{dt^2},$$

for the forces moving the body during the time dt. We may regard each body m of the system as acted upon by parallel forces, then the total force will be equal the sum of these parallel forces. Employing now the sign d, to represent differentials relative to the time, and representing the variations which express the virtual velocity by δ, we have

$$m\frac{d^2x}{dt^2}\,\delta x, \quad m\frac{d^2y}{dt^2}\,\delta y, \quad m\frac{d^2z}{dt^2}\,\delta x$$

for the momenta of the forces

$$m\frac{d^2x}{dt^2}, \quad m\frac{d^2y}{dt^2}, \quad m\frac{d^2z}{dt^2},$$

and for the sum of the momenta

$$\Sigma\left(\frac{d^2x}{dt^2}\,\delta x + \frac{d^2x}{dt^2}\delta y + \frac{d^2z}{\delta t^2}\,dz\right)m.$$

Now let P, Q, R, etc., be accelerating forces acting upon the system and p, q, r their distances, then the differentials δp, δq, δr, etc., represent the variations of the lines p, q, r during the variations δx, δy, δz; but these forces P, Q, R, tend to shorten the lines, therefore their virtual velocities should be written $-\delta p$, $-\delta q$, $-\delta r$, and their moments $-mP\delta p$, $-mQ\delta p$, $-mR\delta r$ and the sum of all these forces will be

$$-\Sigma(P\delta p + Q\delta q + R\delta r + \text{etc.})m.$$

Therefore the sum of all the forces acting upon the body will be

$$\Sigma\left(\frac{d^2x}{dt^2}\delta x + \frac{d^2y}{dt^2}\delta y + \frac{d^2z}{dt^2}\delta z\right)m = -\Sigma(P\delta p + Q\delta q = R\delta r, \text{etc.})m$$

or

$$\Sigma\left(\frac{d^2x}{dt^2}\delta x+\frac{d^2y}{dt^2}\delta y+\frac{d^2z}{dt^2}\delta z\right)m+\Sigma(P\delta p+Q\delta q+R\delta r+\text{etc.})m = 0.$$

"C'est la formule générale de la Dynamique pour le mouvement d'un système quelconque de corps." This formula does not differ from the formula given in his Statics says Lagrange, except in the terms

$$m\frac{d^2x}{dt^2}, \quad m\frac{d^2y}{dt^2}, \quad m\frac{d^2z}{dt^2}$$

which express the accelerating forces. In statics where the acceleration is o, these terms drop out. Therefore this is a general formula applying to statics and dynamics and to solids and fluids. In fact the distinction between statics and dynamics and solids and fluids vanishes except for the difference in substitution in the formulas.

Lagrange then applied this formula to many problems such as, "Sur le mouvement d'un systems de corps libres regardes comme des points et animes par de forces d'attraction." He was the first to make extensive use of the calculus of variations. The idea of this is present in Euler's work in an undeveloped form, but Lagrange was the first to recognize the supreme importance of these ideas and to develop the method of varying arbitrary constants in analysis. He successfully applied this method to the investigation of periodical and secular inequalities of any system of interacting bodies. These methods gave beautiful solutions of such intricate problems as the effect of the disturbance produced in the rotation of the planets by external action on their equatorial protuberances. He also determined the first maximum and minimum values for the slowly varying planetary eccentricities, and contributed memoirs on the "Propagation of Sound" on the "Motion of Fluids," on the "Calculus of Variations," and a "Treatise on Functions and Equations." His notes on the Problem of the Three Bodies, on Variations of the Element of Planetary Orbits, on Attractions of Ellipsoids, and on the Moon's Secular Inequality are noteworthy.

Lagrange verified Newton's theory and developed his suggestions much as Newton did those of Galileo. He reduced the whole theory of mechanics to one fundamental formula, and drew clearly the line between physics and metaphysics. After his time we hear no more such fantastic speculations as were set forth by Descartes and Leibnitz.

Dühring in his "Geschicte der Principien der Mechanik," page 305, sums Lagrange's contribution in these words:

"Die Anwendung eines Fundamentalprincips, welches sich für den Calcül eignet, und die grunsätzliche Durchführing der analytischen Entwicklungen als der Haupt eitfadens für die Verbindung aller Wahrheiten der rationellen Mechanik zu einem einheitlichen System,—das sind die beiden Haupteigenschaften, durch welche sich die Behandlungsart Lagranges auszeichnet." *I. e.*, The application of a fundamental principle adapted to the calculus and the consistent utilization of analysis as his main guide for the combination of all the truths of rational mechanics into a unified system, these are the two points which distinguish Lagrange's method.

Laplace, Simon Pierre, Marquis De (1749–1827).

The genius of Lagrange was at its best in generalization and abstraction and he brought his mind to practical physical problems with difficulty. It was not so with his contemporary Laplace, who was gifted with shrewd practical sagacity in addition to the wonderful mathematical power which won for him the title of the "Newton of France."

He applied himself especially to the great problems of developing an analytical exposition of celestial motions and perturbations, based upon the law of gravitation, and he spent his life in tracing the consequences of the law of gravitation as applied to the solar system.

The solar system does not consist of several bodies, but of a crowd of them traveling about the sun, many of them attended by satellites; thus the complication of attractions is evident. Again the motion of a planet at any time depends not merely upon its relative position with reference to the sun, but also upon the position of the other planets and of its own satellites. Added to this is the difficulty that no planet is where it seems to be, owing to the effects of atmospheric refraction and of the finite velocity of light. The magnitude of the task that Laplace set himself is appalling.

Yet he produced in his "Mécanique Celeste" a work in which the whole theory of planetary motions is investigated, and which offers a complete solution of the great mechanical problem presented by the solar system. It was his constant endeavor to "bring theory to coincide so closely with observation that empirical equations should no longer find a place in astronomical tables." His work is based on the Principia of Newton, which he translates into the language of the calculus, and carries forward and completes so as to produce a mechanical theory of celestial motions.

The "Mécanique Celeste," in five volumes, gives a full analytical discussion of the solar system. The first two give methods for calculating the motions of translation and rotation of the planets, determining their figures and solving tidal problems. The third and fourth volumes contain applications of these formulæ and astronomical tables. The fifth volume is historical. The work is a complete treatise on physical astronomy. The "Exposition du système du Monde" is the "Mécanique Celeste" in popular form without the analysis. The results only are given and the nebular theory is propounded.

Laplace's special contributions to the notation of mechanics are the Laplace Coefficient and the Potential Function. In the course of his work of investigating the figure of a rotating fluid mass, the stability of Saturn's rings, etc., he came upon expressions for the attraction of an ellipsoid involving an integration, which he could not solve. He discovered however that the attracting force in any direction could be obtained by the direct process of differentiating a single function. He

8

was then able to translate the forces of nature into the language of analysis so that he could consider also by mathematical analysis the phenomena of heat, electricity and magnetism.

The function V which was named the Potential Function by Green and Gauss about 1840 is defined as the sum of the masses of the molecules of the attracting bodies divided by their respective distance from the attracting point.

In general terms m being the mass, and r the distance from the attracting point, we have

$$V = \text{Limit}\frac{\Sigma\Delta m}{r},$$

or

$$\Delta m = 0,$$

if ρ is the density of the body at the point x, y, z and α, β, γ the co-ordinates of the attracted point

$$V = \int\int\int \frac{\rho dxdydz}{[(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma^2)]^{\frac{1}{2}}},$$

the limits of the integration being determined by the form of the attracting mass. Therefore V is a function of α, β, γ, that is, it depends on the position of the point, and its several differentials furnish the components of the attractive force. As the integrations did not usually give V in finite terms, Laplace introduced (1785) the partial differential equation

$$\frac{\delta^2 V}{\delta\alpha^2} + \frac{\delta^2 V}{\delta\beta^2} + \frac{\delta^2 V}{\delta\gamma^2} = 0 = \nabla^2 V,$$

Since known as Laplace's equation. Here ∇^2 is called the operator. This equation forms the basis of all Laplace's research in attractions and opened up the whole field of *potential*. This equation is now used in every branch of physical science.

The quantity $\nabla^2 V$ may be viewed as the measure of the concentration of V. Its value at any point indicates the excess of the value of V at that point over its mean value in the neighborhood of the point. This potential function laid the foundation of the mathematical development of heat, electricity and magnetism.

The form in which Laplace first gave his equation, "Recherches sur l'attraction des Spheroides homogenes" in Divers Savans, v. 10, 1873, is, in the polar co-ordinate form,

$$\frac{d\left\{1-\mu^2\right\}\dfrac{dV}{d\mu}}{d\mu}+\frac{1}{1-\mu^2}\cdot\frac{d^2V}{d\omega^2}+r\,\frac{d^2(rV)}{dr^2}=0,$$

where μ is substituted for the cos θ.

If two points in space are determined by their polar co-ordinates r, θ, ω and r', θ', ω', and T be the reciprocal of the distance between them expressed in these co-ordinates, then

$$T=\{r^2-2rr'\,[\mu\mu'+\sqrt{1-\mu^2}\,\sqrt{1-\mu'^2}\cos(\omega-\omega')]+r'^2\}^{\frac{1}{2}},$$

where μ and μ' represent the cos θ and cos θ'.

If this expression be expanded into a series of the form

$$\frac{1}{r'}\left(P_0+P_1\frac{r}{r'}+P_2\frac{r^2}{r^2}+\cdots P_a\frac{r^i}{r'}i\cdots\right),$$

where P_0, P_1, P_a are known as Laplace's coefficients of the orders 0, 1, . . . a, these are found to be rational integral functions of μ and μ', of $\sqrt{1-\mu^2}$ cos ω and $\sqrt{1-\mu'^2}$ cos ω and $\sqrt{1-\mu^2}$ sin ω and $\sqrt{1-\mu'^2}$ sin ω or of the rectangular co-ordinates of the two points divided by their distances from the origin. The general coefficient P_a is of a dimensions and its maximum value Laplace shows to be unity so that the above series will converge if r' is greater than r. He proves that T satisfies the differential equation

$$\frac{d(1-\mu^2)\dfrac{dT}{d\mu}}{d\mu}+\frac{1}{1-\mu^2}\cdot\frac{d^2T}{d\omega^2}+r\frac{d^2(rT)}{dr^2}=0,$$

and if for T the expanded form is substituted we obtain the general differential equation of which Laplace's coefficients are particular integrals

$$\frac{d(1-\mu^2)\dfrac{dPa}{d\mu}}{d\mu}+\frac{1}{1-\mu^2}\cdot\frac{d^2Pa}{d\omega^2}+a(a+1)Pa=0.$$

Laplace's theorem of these functions is to the effect that if

Expressions that satisfy this are called Laplace functions. Y and Z be two such functions, i and i' being whole numbers and not identical then

$$\int_{-1}^{1}\int_{0}^{2\pi} Y_i Z_{i'} d\mu d\omega = 0.$$

The great value of these functions in physical research depends on the fact that every function of the co-ordinates of a point on a sphere can be expanded in a series by Laplace's functions. They are therefore useful in mechanics in researches in which spheres figure, as in the problem of the figure of the earth, the general theory of attraction, and in electricity and magnetism.

Laplace also published in 1812 his "Theorie analytique des Probabilities," an exhaustive treatment of the subject of probability.

It cannot be said of Laplace that he created a new branch of science like Galileo or Archimedes, new principles or a radically new method like Newton, Leibnitz, or Descartes. His work was one of verification and formulation of known ideas into grand generalizations. He possessed a genius for tracing out the remote consequences of the great principles already developed, and he brought within the range of analysis a great number of physical truths which it did not appear probable could ever be brought subject to laws of mechanics. His great contribution was the invention of the potential function in analysis, which, as developed by him and later by Green, Gauss and Lord Kelvin, brought fluid motion, heat, electricity, and magnetism under the dominion of analytical mechanics.

REFERENCES.

Mécanique Analytique. Paris, 1788.
Mécanique Analytique. Paris, 1811.
Exposition du Systeme du Monde. Paris, 1873.
Mecanique Celeste. Translated by Bowditch.
Kelvin, General Integration of Laplace's Differential Equations of Tides.
Dühring, Geschichte der Principien der Mechanik.
Todhunter, Treatise on Laplace's Functions.
Mach, The Science of Mechanics.
Williamson, Treatise on Dynamics.
Todhunter, History of the Mathematical Theory of Attraction.
Thomson and Tait, Treatise on Natural Philosophy.

5. Recent Contributions.

The Contribution of Louis Poinsot (1777–1859).

The contribution of Poinsot to the science of mechanics is one of method rather than of principle. In fact, since the time of Lagrange and Laplace no radically new principle in the science of mechanics has been brought forth, with the exception of the principle of conservation of matter and of energy.

Poinsot's work is set forth in two volumes: "Les Elemens de Statique" and "Theorie Nouvelle de la Rotation des Corps." He follows Newton's method, and builds the science on force, mass, and acceleration as fundamental concepts, but in his exposition the notion of *couples*, *i. e.*, pairs of parallel forces acting on the same body in opposite directions has a prominent part. This idea of a couple was now new; Poinsot did not originate it. It follows from the principle of moments as set forth by Varignon in 1687, but nothing worth mentioning had been made of the idea till Poinsot based a system of mechanics on it, in his Elemens de Statique in 1803. Perhaps no idea in mechanics is so easily comprehended, so useful and so fruitful in the presentation of equilibrium of rigid bodies. But it does not express the historical development of the science. Once mechanics had been developed, it was easy to formulate a system of mechanics by the idea of the couple, but as a rational primitive conception, the idea of equilibrium established in this way does not appeal to the mind.

Poinsot says, in the preface of the "Elemens": "Dans la solution mathematique des problemes, on doit regarder un corps en equilibre comme s'il etait en repos; et reciproquement, si un corps est en repos, on sollicite par des forces quelconques, on peut lui supposer appliquees telles nouvelles forces qu'on voudra, qui soient en equilibre d'elles-memes, et l'etat du corps ne sera point change. On verra bientot de nombreuses applications de cette remarque." One may regard a body in equilibrium as if at rest, and one may regard a body at rest as being so, because the forces applied to it balance each other. One may assume various other pairs of forces applied to the body and it will still remain at rest. This idea has many useful applications.

He then develops the idea of a couple and sets forth a number of theorems on couples from which he evolves the theory of the simple machines. He says: "Nous reduirons les machines simples a trois principales que l'on peut considerer si l'on dans l'ordre suivant en regard à la nature de l'obstacle qui gene le mouvement du corps: le levier le tour et le plan incline." The simple machines may be reduced to three principles according to the nature of points considered as fixed, viz: the lever, the screw and the inclined plane.

In the first, the obstacle or impediment is a fixed point; in the second, it is a straight line; in the third, it is a fixed plane. From these he develops geometrical theorems on the simple machines.

In general, Poinsot's method is distinctly his own development of a synthetic mechanics, based on Newton's ideas. He does not use the calculus, but develops the whole system by a judicious choice of fixed points and by the action of couples. He gives a self-contained exposition of the science which is useful rather as a practical text-book than as a system for advancing the science. The Theorie Nouvelle de la Rotation des Corps treats of the motion of a rigid body by geometry and shows that the most general motion of such a body can be represented at any instant by a rotation about an axis combined with a motion of translation parallel to the axis, and that any motion of a body, of which one point is fixed, may be produced by the rolling of a cone fixed in a body on a cone fixed in space. This enables one to picture the motion of a rigid body as clearly as the motion of a point. The previous treatment of the motion of such a body had been analytical, and gave no mental picture of the moving body.

Poinsot's exposition of statics and of rotation by the action of couples about arbitrarily chosen fixed points, lines, or planes, is valuable as offering ready practical conceptions of mechanical action for every-day use. It is just such a system as one would expect a professor in a technical school to develop for the use of students who were preparing for professional work rather than for research. The diagrams demonstrate the theorems so as to make the proof almost axiomatic and intuitive. His theorems are to be found to-day in modern text-books and are of service to the mechanical and civil engineer.

Among his memoirs are contributions on: "Sur la composition des moments et des aires." "Sur la geometrie de l'equilibre et du mouvement des Systemes." "Sur la plan invariable du systeme du monde." His Mechanics is valuable for its ready practical methods, rather than for new contributions to the science.

The Contributions of Simeon Denis Poisson (1781–1840).

Poisson, the distinguished young contemporary of Laplace and Lagrange, was their equal in mathematical analysis and their superior in grasp of physical principles. A large number of memoirs, on a wide range of scientific subjects, testify to his ability. In some of these he corrected errors in the work of Laplace and Lagrange.

Poisson applied himself particularly to mathematical physics. He explored heat, light, electricity and magnetism by analysis and originated the method of investigation by "potential." He evolved the correct equation for potential

$$\nabla^2 V = -4\pi\rho$$

in place of Laplace's equation

$$\nabla^2 V = 0.$$

This equation now appears in all branches of mathematical physics, and, according to some writers, it follows that it must so appear from the fact that the operator ∇^2 is a scalar operator. Indeed it may be that this equation represents analytically some law of nature not yet reduced to words.

Poisson's work, "Traité de Mécanique" (1853), is an excellent exposition of rational mechanics by the method of the calculus. It proceeds logically from the definitions of "corps," "masse" and "force," and a definition of Mechanics "la science qui traite de l'equilibrium et du mouvement des corps" through statics and dynamics, section by section. Though it contains some variations in mathematical presentation, it contains no new principle.

His work on the theory of Electricity and Magnetism and his "Théorie Mathematique de la Chaleur," 1835, present methods by which nearly all physical phenomena may be explained in terms of mathematical mechanics. With this the science of mechanics approaches its highest development. From the time of Poisson up to the present, a number of investigators have worked over the field and developed the applications of known principles and methods. Among them must be mentioned:

Fourier, Theorie analytique de la chaleur, 1822.
Gauss, De figura fluidorum in statu æquilibrie, 1828.
Poncelet, Cours de mecanique, 1828.
Belanger, Cours de mecanique, 1847.
Mobius, Statik, 1837.
Coriolis, Traite de Mecanique, 1829.
Grausmann, Ausdehnungslehre, 1844.
Hamilton, Lectures on Quaternions, 1853.
Jacobi, Vorlesungen über Dynamik, 1866.
Joule, J. P., Scientific Papers, 1887.

As a result of the earnest labors of these and others, and more particularly by the patient research of those mentioned below, the nineteenth century saw the establishment of the great mechanical principle of conservation, the most unifying and fruitful of all scientific dogmas. It is the result of the accumulated experience of many inquirers rather than the achievement of any individual.

THE LAW OF CONSERVATION.

In 1775, the French Academy declined to consider any further devices for obtaining "perpetual motion," but it was not till one hundred years later, about 1875, that the generalizations known as the Conservation of Matter and the Conservation of Energy, or the Law of Conservation came to be generally admitted after long experiment and careful study.

The principle of the Conservation of Matter was established about 1780 by Lavoisier, (1743–94), as a result of a series of experiments with the chemist's balance which indicated that the mass of a given quantity of matter remains constant regardless of change of state or of chemical combination.

The principle of conservation of energy was of slow growth. The idea of conservation in nature seems to have been dimly felt as far back as the time of Descartes (1596–1650). Newton, also, seems to have had an idea of it, though his development of mechanics by the concepts of work, force and distance, blinded him to the appreciation of the measure of activity by energy. Still in the scholium to his third law, we read: "If the action of an agent be measured by the product of the force into its velocity, and if similarly the reaction of the resistance be measured by the velocities of its several parts multiplied into their several forces, whether they arise from friction, cohesion, weight or acceleration, action and reaction in all combination of machines will be equal and opposite." It is probable that the popularity of the Newtonian exposition of mechanics from the point of view of force and work, had a tendency to delay the establishment of this principle of conservation. The concept of Energy was foreign to Newton's mechanics.

The principle was rather a slow development of the Huygenian idea of energy and it came to the fore, with the recognition of a relation between mechanical energy and heat. The idea that heat is a form of energy for which there is an exact mechanical equivalent was first suggested about 1798, by the experiments of Count Rumford on the heat resulting from the boring of cannon and by the experiments the following year, of Sir Humphrey Davy on melting ice by friction. This conception was at variance with the generally held hypothesis that heat was of the nature of a material fluid.

The idea languished till 1842, when Julius Robert Mayer began experimental research on the subject. Choosing as the unit of heat, the quantity necessary to raise one gram of water at 0° C., one degree centigrade, commonly called a "calorie," and for the unit of work, one gram lifted one meter or a "gram-meter," the determination of the number of gram-meters that are equivalent to a calorie in energy was stated by Mayer as 365 from his experiments on the heat evolved in compressing air.

In 1843 J. P. Joule (1818–89) undertook the investigation of the subject and invented a variety of apparatus for determining the dynamical equivalent of heat and among other forms the common laboratory method of descending weights turning paddle wheels in a vessel of water, the temperature of which is determined by thermometers. The subject now came up for thorough investigation and discussion by scientists. Helmholtz maintained the principle in "Ueber die Erhaltung der Kraft," 1847, and Rankine, Kelvin, Clausius and Maxwell contributed either experimentally or theoretically to its establishment.

It is worded in various ways, one form being: In any system of bodies the energy remains constant during any reaction or transformation between its part. It is also stated as: "The energy of the universe is constant."

In 1850 Joule obtained his value 423.5 gram-meters for the dynamical equivalent of heat which for two decades was the accepted value. By 1860 research had verified this figure by transformations of energy through mechanical, electric, magnetic and chemical transformations in sufficient number to warrant the acceptance of the principle of conservation of energy. Prof. Rowland in 1879 made a series of very careful determinations of the dynamical equivalent of heat using Joule's stirring or paddle apparatus, and finally gave the value 425.9 for water at 10° C.

This principle is, as Maxwell says, "the one generalized statement which is found to be consistent with fact, not in one physical science only but in *all*. When once apprehended, it furnishes to the physical inquirer a principle on which he may hang every known law relating to physical actions, and by which he may be put in the way to discover the relations of such actions in new branches of science." He states the principle as follows: "The energy of a system is a quantity which can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible." The total energy of a closed system is invariable quantity.

Whether the energy of a system is partially in the kinetic and partially in the potential form, whether the energy exists as potential energy of arrangement of the gross parts of a system, or as molecular energy, or electrical energy, or as kinetic energy of moving masses, or of moving molecules, or of vibrations of the ether or of electrical currents, the total quantity of energy in an isolated system is constant.

We have no acquaintance with "absolute energy" or of energy apart from matter. Our knowledge is limited to energy changes in matter. Work done upon a body or a system increases its energy, or work done by it upon another body confers energy upon it. If we do work upon a body weighing 100 lbs. so as to raise it vertically 5 ft. we store 500 ft. lbs. of energy in it, which is said to be in the "potential" form. The mathematical expression of energy always requires two factors. For instance, in doing mechanical work we may measure the energy by the product of the force times the distance, $F \times S$, or if the work has produced kinetic energy we measure it by the mass of the body multiplied by the square of the velocity, *i. e.*, $mv^2/2$. In case the mechanical work is transformed into heat the factors become the specific heat and the rise in temperature. If the heating is produced by a transformation of electrical energy, the electrical energy is measured by the quantity of electricity and the electromotive force.

From the principle of conservation have been evolved the three principles of thermodynamics or of energetics which are commonly listed as:

(1) the conservation of energy;

(2) the distribution of energy or the principle of Carnot;

(3) the law of least action.

The second principle is given by Clausius in the form: "Heat cannot of itself pass from a colder body to a warmer one." Lord Kelvin put it thus: "It is impossible, by means of inanimate material agencies to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest surrounding objects."

This was later generalized and put into the form: The transfer of energy can only be effected by a fall in tension. This is the principle of Carnot and signifies that energy always goes from the point where the tension is high to the point where it is low. This applies not only to heat but to all known forms of energy.

If we imagine a system of bodies taken at random in various conditions of temperature, electrification, etc., they will not remain as thrown together, but a readjustment, with transferences and transformations of energy will begin, until one of the factors of the energy of all the bodies has the same value or intensity in all parts of the system.

That is, if the electromotive force or the temperature is the same in all parts of the system, no transference takes place; or, if for the kinetic energy, the velocity is the same, there is no change; but whenever there is a difference there will follow a change within the system. The third principle of thermodynamics says that these changes always follow a path which requires the least effort. This is sometimes named Hamilton's principle. With these theories of readjustment and flux of energy the occasion and character of the various changes or phenomena of the material world may be schematized.

It is worthy to note that no one has succeeded in exactly and completely reversing a series of natural processes. There is always a loss of energy usually as heat, in any series of transferences or transformations of energy. The researches of Clausius and Planck seem to prove that there is a constant "degradation" of energy or a reduction to the condition of a dead level. Without tension or difference in potential there is no transmission of energy, nor can there be any work done.

Having attained then, the mechanical conception of energy and the principles of conservation, we come into possession of a unified theory and a workable scheme of antecedents and sequences of the gross phenomena of nature, which now become a subject of calculation by mathematical analysis as formulated in Analytical and Celestial Mechanics.

Granted a certain quantity of energy in a material system, the conditions of its transfer and transformation are now become a matter of mathematical calculation, and the concomitant gross phenomena may be predicted with certainty and precision. The great principle of conservation of energy is a wider generalization than the Newtonian mechanics. It has enabled us to advance our explanation of the motion-phenomena of the universe, but we are still far from explaining all phenomena by Mechanics. The result of recent efforts to extend the science so as to explain the minuter and more subtle phenomena of the universe will now be briefly commented upon.

6. The Ether. Energy. Dissociation of Matter.

The nineteenth century saw the general acceptance of Lavoisier's adage, "Nothing is created, nothing is lost." With the gradual establishment of the idea of conservation came an enthusiastic endeavor to unite the various separate sciences into Science by means of the concept of energy. Energy being conceived as a measure of activity and the quantity of energy being considered invariable, it is logical to expect that all the phenomena of the universe might be co-ordinated by this idea.

Mechanics which had developed the concept of energy and a series of mathematical equations expressing its relations, from a study of the gross motion phenomena of the world, had arrived at what appeared to be a universal law. And now the various separate chains of phenomena which had been linked together by the chemist, the physicist, the botanist and the biologist were to be welded into one Science by the principles of mechanics. The chemists had been working toward the idea of conservation for nearly a century and when chemistry and mechanics came into accord upon the idea of conservation, it was felt that it must fit the other sciences too, and that it was the key to nature's secrets.

A review of the scientific beliefs of twenty-five years ago reveals a faith in the duality of natural phenomena. They were conceived as the result of the action of indestructible energy through indestructible matter which was conceived as floating in an all pervading medium called the ether of space.

This medium was conceived as penetrating and pervading all matter.

The idea of an ether of space appears to be very old. The term is derived from the Greek word æther, meaning the brilliant upper air. The hypothesis in later times was the result of the logic that demanded a medium to transmit light and heat through interplanetary space and through a vacuum. Hence it was at first called the light-bearing or luminiferous ether.

Fresnel (1788–1827), the French physicist, in his undulatory theory of light first gave this hypothesis definition. Later Faraday (1791–1867) likewise postulated a medium in connection with his researches in electricity and magnetism and suggested that perhaps one and the same medium would serve for both light and electricity. The researches and calculations of numerous investigators among whom Maxwell was prominent finally gave decision in favor of one medium or ether, possessing certain characteristics.

Being a purely arbitrary hypothesis the ether could and soon came to be endowed with such properties as were called for by the logic of the situation, and these properties were altered from time to time as seemed necessary. The ether was declared to possess *inertia*, because time was required for the propagation of light through it. It was conceived as having *density* and *elasticity* by analogy with matter, and it was pictured as an "elastic jelly." In this medium, waves varying in length from miles to less than two millionths of a millimeter were conceived as explaining various phenomena of light, heat, electricity and magnetism. Though nothing is positively known of the existence or structure of the ether, this convenient assumption has been developed with great definiteness.

Once this hypothesis was established Mechanics entered upon a new phase of development. It was called upon to deal with molecular and atomic energy and invited to explain by its principles the minute phenomena of light, electricity and biology. In this it relied upon the unifying power of the law of conservation and the license to warp and model the suppositititious ether to the exigencies of the occasion.

How far this has been successful can be but briefly considered here. It soon became apparent that the molecule or smallest portion of physical matter, sometimes pictured as bearing to a drop of water the ratio that a golf ball bears to the earth, must give up its simplicity as a dense hard sphere and become constituted of at least several atoms of various densities to comply with the chemist's notions of elementary and compound substances.

Before long, these atoms had assumed the complexity of solar systems and were conceived as composed of thousands of particles or electrons in rapid motion, and as being of many varieties. Here we see at work the familiar old primitive notions of division, moving particles and pictorial representation. In the hands of such investigators as Fizeau, Crookes, Kelvin, Lodge, Le Bon, Michelson, Morley, Rayleigh, Ramsey, Roentgen, J. J. Thomson, Rutherford and others, the method has been applied in linking up, by the principles of gross mechanics a variety of minute phenomena. It has led experimental research through numerous novel and remarkable investigations in light, heat and electricity from which much is expected.

With the discovery of the X-Rays by Roentgen in 1895, and of radioactivity by Becquerel and the Curies in 1898, and with the discovery by J. J. Thomson that the passage of these activities through the air makes it a conductor of electricity, new conceptions arise. The air as we commonly know it, is a non-conductor of electricity but "ionized" air produced by radioactivity, or by the emanations from such substances as radium, thorium, and polonium, is a conductor. It soon became evident that a great many bodies in nature are spontaneously active and are constantly giving out emanations.

Investigation showed that these emanations have the power of dissociating a gas, or of breaking it up into particles, comparable with hydrogen atoms, and particles approximately one thousandth as large, called electrons. The velocity of these particles approximates that of light and their total mass or inertia appears to be due to an electric charge in motion. In other words the one characteristic invariable property of matter, viz: mass, is explained as an electric charge in motion.

Larmor, in his "Ether and Matter," says the atom of matter is composed of electrons and of nothing else. This conception builds matter of electricity in motion, though it is a question as to whether this is a simplification or a complication of theory.

The question as to where these electrons get their motion, or what is the origin of the energy which expels these emanations with such terrific velocity, has been met by a mechanical hypothesis of the atoms as whirling "solar systems" of thousands of electron-satellites, some of which, when equilibrium is disturbed, fly off tangentially with great velocities. This is practically saying that molecules and atoms of matter on their disruption or dissociation set free energy. Experiments on radioactivity show that a gram of radium will raise the temperature of 100 grams of water 1° C. an hour without perceptible loss of weight on the chemist's balance. But the researches of Prof. Crookes and Dr. Heydweiller,[1] estimate the duration of a gram of radium at about 100 years after which there is no longer any radium, therefore a quantity of highly heated water may be left as a result of its emanations if we conceive it to act upon water. Here matter disappears and energy in the form of steam pressure appears in exact ratio.

This brings us face to face with a contradiction of the law of conservation as we have stated it. We have matter fading into the ghost of matter losing its one distinguishing unalterable characteristic, namely, *mass*, and liberating an enormous quantity of energy in the process. From a mechanical point of view this is a contradiction in terms but the advance guard on the skirmish line of science necessarily uses the terms that are at hand with various mental reservations and modifications until nomenclature can be revised and remodeled. With every advance in Science there is inevitably a period of temporary anarchy in theory and terminology. The concepts of energy and electricity appear to be about to go through some such period of transformation as has happened with the term force.

We find ourselves now on the threshold of the realization of the dream of the alchemist. These X-rays, emanations,

[1]P. 237.
[2]*Phys. Zeitschrift*, October 15, 1903.

ions, electrons and electricity appear to be phases of the dematerialization of matter, stages in the breaking down of matter into intra-atomic energy. As Professor de Heen of Liege says, "it seems we find ourselves confronted by conditions which remove themselves from matter by successive stages of cathode and X-ray emissions and approach the substance designated as the ether."

Further researches indicate that electricity is one of the forms of energy that result from the breaking up of atoms, that it is composed of these imponderable electrons, the ghostly emanations of fading matter which themselves have been pictured as but minute whirls in the all pervasive ether. We come here to a new conception, matter is conceived as built up of electrons, pictured as little whirl-pools in a fundamental ether of which the universe is composed. However this may be, we are made acquainted with stores of energy and activities as little known as electricity was before Volta's day. The establishment of the fact of the dissociation of matter opens up unsuspected and inconceivable sources of energy. The energy liberated from the partial dissociation of a tub of water would probably equal that of all the anthracite coal fields of America.

This theory hints at an explanation of some of the mysterious activities of vegetable and animal life. The researches of biological chemistry are just beginning to reveal some of the secrets of the flux and reflux of intra-atomic energy in highly complicated and unstable compounds and the incidental liberation of (electrical) energy. The theory also offers suggestions as to the character of allotropy, catalytic action, diastases, toxins and protoplasmic action. These minute phenomena of nature are motion-phenomena and as such come within the purview of mechanics, but in the development of a theory of the grosser phenomena they have had scant attention. It may be that the laws of gross mechanics do not apply here exactly, at any rate it seems that there is enough suspicion of mutation of matter and flow of energy to put the law of conservation on the defensive.

The most radical contradiction of the now commonly accepted doctrine of conservation is that given by Dr. Gustave

Le Bon in his "Evolution of Matter," 1905, from which the following summary is taken.

"1. Matter, hitherto deemed indestructible, vanishes slowly by the continuous dissociation of its component atoms.

"2. The products of the dematerialization of matter constitute substances placed by their properties between ponderable bodies and the unponderable ether—that is to say between two worlds hitherto considered as widely separate.

"3. Matter, formerly regarded as inert and only able to give back the energy originally applied to it, is on the other hand, a colossal reservoir of energy—of intra-atomic energy—which it can expend without borrowing anything from without.

"4. It is from the intra-atomic energy, manifested during the dissociation of matter that most of the forces in the universe are derived, notably electricity and solar heat.

"5. Force and matter are two different forms of one and the same thing. Matter represents a stable form of intra-atomic energy; heat, light, electricity, etc., represent unstable forms of it.

"6. By the dissociation of atoms,—that is to say, by the dematerialization of matter, the stable form of energy termed matter is simply changed into those unstable forms known by the names electricity, light, heat, etc.

"7. The law of evolution applicable to living beings is also applicable to simple bodies; chemical species are no more invariable than are living species."

These are bold generalizations made from comparatively scanty experimental data on very minute and delicate phenomena, and they are not unchallenged. But they suggest a new departure and a new phase of development in mechanics and hint at marvels until now undreamt of.

As to the possibility of producing energy for industrial purposes by breaking down or using up matter and thus turning it into energy, the expectation is certainly as bright as was the prospect, that Volta's early electrical experiments with frogs' legs and a copper wire would ever lead to the operation of heavy railroad trains by electricity or to the transmission of the voice from city to city, by wire, or of "wireless messages" from mid-ocean to shore.

The wonders of aerial telegraphy and telephony are the result of careful investigation and study in this new field of what might be called the mechanics of the ether. When the "activities in the ether" are more thoroughly understood we may expect greater wonders. It is to be noted that it is not always the most intense action that will produce a desired result. A thunder clap will not move a tuning fork to vibration, whereas the vibration of a violin string will do so if of the proper key. A spark is ridicuously inadequate as compared with the explosion of energy it may cause. The simple striking of a phosphorus-match by moving it with a velocity of about ten feet a second, serves to set up disturbances which have a velocity of 186,000 miles a second.

Atomic energy, of the existence of which there seems to be no doubt, is practically inexhaustible in amount, as simple calculations show. The energy that would flow from the dissociation of a one cent copper coin is equal to the energy of 1,000 tons of coal applied in the production of steam. Mechanics has brought us from the dim gropings of the Stone Age, for "more power to the arm," to an outlook upon an immense universe of ceaseless energy. When mechanical contrivance shall have caught up with, and exploited this vision we may expect a conquest of power that will accomplish inconceivable wonders.

This then is the fruit of fifty centuries of patient endeavor in mechanics, of 2,000 years of geometrical mechanics and 200 years of analytical mechanics. It is the heritage of the patient fidelity and stern integrity of the great inquirers and their numerous minor coadjutors, and it presages a greater and more marvellous harvest of enlightenment and benefaction for the future. The indomitable courage and patience of these searchers for ultimate and invariable truth have emancipated the race from much of the incubus of the superstitious fetishism, and from some of the drudgery of daily life, and they point prophetically to greater conquests to come. But in the words of one of the eminent sages[1] of the science, all have thus far been as little children picking pebbles on the shore, while the great ocean of the unknown glooms beyond. The words of Laplace are still all too true, "What we know is little, what we do not know, immense."

[1]Sir Isaac Newton.

PART IV.

CONCLUSION.

The history of the science of mechanics has now been traced in outline. We have noted its aspirations; we must now note its limitations. Science is human experience tested and arranged in order. It is not its purpose to offer a philosophy of the universe, nor is it essentially in conflict with religion. It seeks, rather to co-ordinate experiences into a systematic theory of relations, of causes and effects. The discovery of natural truths and the extension of the field of knowledge by a process of correlation, rejection, revision and verification is its province.

We note that the science is a mental resumé of the growing experience of the race, a development founded on many centuries of endeavor in the arts and trades. It had its origin in the dim past with geometry which evolved from land-surveying as mechanics did from the trades. The science is essentially the product of European thought. In the nature of things its development consisted in abstracting from the numerous phenomena of nature the constant elements, this method obviously indicating itself as the path of progress. Once the abstractions of form and position were realized, study of forms and positions led to the development of a geometry of measurement and an arithmetic. Until this point is reached not much can be expected in physical science, for the spur of progress is the question "how," and no satisfactory answer can be given to it until a system of measurements is developed.

When once the abstract conceptions of form and position are firmly established and a method of measurements devised, then the conditions and circumstances of change of position and of change of form and size present themselves as questions of possible investigation.

Even after the Greeks had developed geometry, their ideas

TABULAR VIEW OF THE DEVELOPMENT OF THE SCIENCE OF MECHANICS.

Date.	Contribution.	Published in	Author.	Period.
B.C.				
10000	Common mechanical experiences of the lever, wedge, bow and arrow, etc.	Archæological studies of cave and lake dwellings.	(Abbott; Munro; Petrie.)	(Prehistoric.)
3500	Mechanic trades developed in Babylonia, Egypt and Phœnicia; experience with the simple machines in rudimentary form.	Cuneiform and hieroglyphic inscriptions; wall pictures and drawings on pottery.	(Maspero; Perrot-Chipiez; Botta; Layard.)	(Dawn of history.)
2500	Rudimentary arithmetic.	Ahmes papyrus of the British Museum.	Ahmes (transl. Eisenlohr).	Cir. 2000 B.C.
600	Surveying and early geometry.	Manuscripts.	Thales.	Cir. 600 B.C.
500	Arithmetic and geometry	Manuscripts.	Pythagoras.	Cir. 500 B.C.
300	Formal geometry.	"Euclid."	Euclid.	Cir. 300 B.C.
225	Principle of the lever. Center of gravity. Principle of buoyancy. The beginning of Statics.	Equiponderants, etc.	Archimedes.	287–212 B.C.
A.D.				A.D.
900	Arabic numeration. Algebra.	Ilm Aljabr Wa'l Muqabalah.	Ibn Musa	Cir. 900
1500	The Statical moment.	Manuscripts.	Leonardo da Vinci.	1542–1519
1577	The Statical moment.	Mechanicorum Liber.	Guido Ubaldi.	1545–1607
1608	The triangle of forces. Graphic Statics. Elementary Hydrostatics.	Hypomnemata Mathematica.	Stevinus.	1548–1620
1642	Dynamics founded. Theory of falling bodies. Isochronism of pendulum.	Discorsi, etc.	Galileo.	1564–1642.
1644	Algebraic geometry. A system of universal mechanics.	Principia Philosophiæ.	Descartes.	1564–1642.
1662	Hydrostatic theory.	Traité.	Pascal.	1623–1662
1663	Pneumatic theory.	De Vacvo Spatio.	Guericke.	1602–1686
1665	Law of volume and pressure of gases.	Experimenta Physico-Mechanica.	Boyle.	1627–1691
1666	Elementary Hydraulics.	Traite du Mouvement des Eaux.	Mariotte.	1620–1684
1673	Theory of the pendulum. Dynamics of a particle. Centrifugal force.	Opera. Horologium.	Huygens.	1629–1695

Date.	Contribution.	Published in	Author.	Period.
1668	Laws of impact.	Mechanica.	Wallis.	1616–1703
1686	The Laws of Motion. Law of Gravitation. Motion about a center. A Geometrical Exposition of Mechanics. Fluxions.	Principia, 1686. Manuscripts.	Newton.	1642–1726
1687	A complete theory of Statics based on balanced forces. Geometrical proof of statical moment.	Nouvelle Mecanique, 1687.	Varignon.	1654–1722
1700	Concept of Energy. The Calculus method.	Acta Eruditorum.	Leibnitz.	1645–1716
1717	Principle of Virtual Velocities.	Letter to Varignon.	J. Bernoulli.	1667–1748
1757	Principle of Least Work.	Œuvres, 1752.	Maupertuis.	1698–1759
1740	Development of Hydrodynamics.	Hydrodynamica, 1738.	D. Bernoulli.	1700–1782
1743	D'Alembert's Principle. Analytical Mechanics.	Traité de Dynamique, 1743.	D'Alembert.	1717–1783
1744	System of Co-ordinate Analysis.	Methodus, 1744.	Euler.	1707–1783
1788	Principle of Virtual Displacements. Lagrange's Equation.	Mecanique Analytique, 1788.	Lagrange.	1736–1873
1800	Theory of Potential. Celestial Mechanics.	Mecanique Celeste, 1799.	Laplace.	1749–1827
1803	Theory of Couples. A Theory of Statics based on couples. Rotation theory.	Elemens de Statique, 1803.	Poinsot.	1777–1859
1840	The development of Potential and the application of Mechanics to heat, electricity and magnetism.	Memoirs.	Poisson.	1781–1840
1850	The Law of the Conservation of Matter and Energy.	Scientific papers.	Mayer, Joule, Helmholtz, Tyndall, Kelvin, Maxwell.	Cir. 1870
1900	Ether theory. Electron theory. Dissociation of Matter.	Current publications.	Crookes, Becquerel, Thomson, the Curies, Lodge, LeBon, and others.	Cir. 1900

on natural phenomena are found expressed in such naive statements as those of Aristotle to the effect that all bodies have a place and seek their place; bodies float because of their form; if they move, their motion is either natural or violent; heavy bodies go down because they belong down under the lighter ones; the heavier they are, the farther down they belong and the faster they move to get there! It was not till the abstract concept of force was completely attained to, eighteen hundred years later, and the concomitant circumstances of it studied and measured that these ideas gave place to our modern generalizations.

The measurement of the constant elements in natural phenomena naturally began with bodies near at hand and at rest, with statics and hydrostatics. We find that the great contribution of Archimedes is not so much the rules and methods commonly associated with his name, as the development of a science of measurement and its application in the study of natural phenomena. With this the science of mechanics began.

Here at its very basis we find the abstract concept and the mathematical notion of relativity as fundamental to the science. Those who ask a physics or mechanics without abstractions and mathematics are seeking science without its most essential and useful features. The further review of the science indicates that an evolution of abstract concepts and a development of the application of analysis in connection with them is an inevitable necessity of its progress.

It is not the purpose here to consider metaphysical and psychological questions, therefore we will not speculate upon the probability or possibility of developing a science of mechanics on another basis than the abstract concept linked up mathematically. That we cannot know matter or the phenomena of nature except as mental percepts or concepts is a trite saying; it seems to follow logically therefore that our mechanics must be built up of these elements, however much experience and study may change and elaborate them.

We have seen how the notion of force developed and changed. How at first it was probably anthropomorphic, conceived as the muscular vigor of an invisible demi-god, how it underwent transformation, served a very useful purpose in the development of mechanics and is now discarded in some text-books as an outworn subjective idea to which there is nothing in nature to correspond. It is so with other ideas, and perhaps many of them will continue to go through some such development and transformation. They will be changed, modified or discarded. Mechanics was developed by geometric methods of measurement up to the year 1700; then, the method of fluxions and limits made possible the investigation of quantities whose value is continually changing, and the science made a wonderful advance.

In all mechanical experience there are two conceptions, however, which are constantly present, and form, as it were, the background—Time and Space. They appear to be fundamental and ultimate; being irreducible, they cannot be compared with anything, and are indefinable. With them a third conception variously pictured and called, matter, energy, ether or electron, suffices to form the rational resumé of phenomena which is called mechanics. The riddle of time and space is a question of metaphysics, not of mechanics. Considering it briefly we note there are several points of view. One may hold after the manner of Trendelenberg in his "Logische Untersuchungen," Chap. V, that there exists the conception of motion quite apart from the ideas of space and time, which he derives from it. His endeavors to prove a knowledge of motion prior to any idea of position or of sequence are not convincing. Though the theory is plausible, it is not proven. A second theory due largely to the philosophy of Kant and Hume would make time and space merely modes of perception, ways in which the perceptive faculty distinguishes objects. Though this is not admitted generally as expressing the full truth, it indicates clearly the intimate relation between time and space.

They are bound together in a way which may be pictured by supposing space to represent the breadth of our field of perception, then time would represent its length. Space marks the co-existence of perceptions at a point in time, so time marks the progression of perceptions at a position in space. The two modes combined give us motion as a fundamental way in which we conceive phenomena.

If we admit this, the mode of perceiving things in this way would seem an essential feature of our conscious life. With only the one space mode of perception, we could know only of co-existing things, of number, position and measurement; our science would be limited to arithmetic, algebra and geometry, and the phenomena of motion would not exist for us. We could not conceive of warmth, weight, hardness, etc., for these depend upon sequence, on time. On this theory the perceptive faculty sorts sense impressions by these two modes, as upon a rack or frame-work; and if the simile be permitted one may say both co-ordinates are necessary. Neither infinity of space or of time, or empty time find place on the frame-work, nor have they meaning in the field of perception.

Space and time in this view are modes of perceiving things. They are not necessarily, per se, infinitely large or infinitely divisible, but are essentially relative. The reality of time and space is not a point of discussion in this paper, but this philosophical theory is often involved with the philosophy that denies or is agnostic as to the reality of matter,—regarding sense perceptions under the modes of time and space as the only realitites.

A more rational point of view and one that will appeal to a greater number is the theory that neither denies the existence of matter and motion apart from perception nor affirms that time and space are but modes of thought, and that motion is the concomitant of their relation to each other. According to this theory the reality of moving bodies is not denied, but it recognizes that our knowledge of them is limited to what we may perceive and conceive of under the modes or limitations of time and space. It recognizes our hypotheses and principles as approximations, and questions whether we can ever fully conceive of or comprehend the realities of nature in all their completeness.

The concepts developed with and within these modes of time and space, such as geometrical surface, atom, molecule, force, etc., are not necessarily the realities. Often they are obviously not so. But these terms are very useful in picturing the correlation and sequence of phenomena. The continual shifting and the evolution which we see going on in our scientific concepts indicate that they are approximations, and points the necessity for caution in speaking of them as realities, or of projecting ideal dancing or whirling molecules into the world of the actual.

As to this third conception which, together with time and space, suffices to illustrate the phenomena of nature, it is variously conceived and described as matter, as ether, and as electrons. The first and oldest of these was the idea out of which the others have arisen as later and broader knowledge imposed more precise requirements. The older books on mechanics went along with "corpus" and "moles" until the distinction between weight and mass was made. Then "masses" sufficed until more recent times when matter came to be commonly defined in the text-book by its properties of extension and inertia. The modern view is given in the little book "Matter and Motion," by Clerk Maxwell thus, (page 163): "All that we know about matter relates to the series of phenomena in which energy is transferred from one portion of matter to another till in some part of the series our bodies are affected, and we become conscious of a sensation. We are acquainted with matter only as that which may have energy communicated to it from other matter. Energy, on the other hand, we know only as that which in all natural phenomena is continually passing from one portion of matter to another. It cannot exist except in connection with matter."

In effect, this paragraph defines this third thing as a medium for the storage and communication of energy, without telling what it is or what energy is. Having arrived at "energy" as a convenient conception and being under the necessity of conceiving of it as stored and transmitted, the idea of matter is made to assist to that end.

In the well known Treatise on Natural Philosophy by Thomson and Tait we read (p. 207): "We cannot of course, give a definition of matter which will satisfy the metaphysician, but the naturalist may be content to know matter as that which can be perceived by the senses or as that which can be acted upon, or can exert force." This definition, like the first, is of a dual character indicating matter as the medium of the action of force instead of energy. Either of these definitions will serve for a development of mechanics from the view of energy or of force. With either of these premises granted a logical mechanics is possible.

If we consult Tait's "Properties of Matter" (pp. 12–13 and pp. 287–91), we read: "*We do not know, and are probably incapable of discovering what matter is*," and, "*The discovery of the ultimate nature of matter is probably beyond the range of human intelligence*." This is at least decisive, but some would probably say it is unnecessarily blunt and discouraging.

The idea of matter, proving inadequate with the advance of the science, and the application of molecular mechanics in the study of light, electricity and heat, the theory of the ether as a perfect fluid medium and a perfect-jelly medium was introduced. The jelly-theory of the ether has undoubtedly been of value in simplifying many of our views of physical phenomena, but not being entirely satisfactory, the "vortex atom" and "vortex ring" in the ether were invented. This was followed by Kelvin's "ether-squirt." From periodic variations of the rate of squirting as influenced by the mutual action of groups of squirts, he was able to picture and deduce many of the phenomena of chemical action, cohesion, light and electro-magnetism.

A more recent theory is the corpuscular or electron theory as expounded by J. J. Thomson in "The Corpuscular Theory of Matter," 1907, which supposes that "the various properties of matter may be regarded as arising from electrical effects." On page 2 of this volume we read: "This theory supposes that the atom is made up of positive and negative electricity. A distinctive feature of the theory—the one from which it derives its name—is the peculiar way in which the negative electricity occurs both in the atom and free from the atom." He supposes that the negative electricity always occurs as exceedingly fine particles called corpuscles, and that all these corpuscles, whenever they occur, are always of the same size and always "carry the same quantity of electricity." "Whatever may prove to be the constitution of the atom we have direct experimental proof of the existence of these corpuscles." This theory has the advantage of being able to explain electrical metallic conduction. It explains mechanical inertia as the self induction of an electric current, and mass on the basis of the velocity of the corpuscles. On this theory matter is conceived of, as in part at least identical with electricity, and the properties of matter are explained as electrical effects. But this still leaves a third concept, electricity in addition to time and space. It is interesting to note that in the exposition of this theory, analysis and the elementary mechanical concepts of velocity, mass and energy, etc., are used to attain to this new idea. This illustrates the evolution of new concepts from the old, as the path of advance in mechanics, and in all science.

Yet even these closer approximations cannot be regarded as realities. They indicate mechanical actions which may be close approximations of the reality but there is no ground for calling them identities.

Dr. Ernst Mach says,[1] "purely mechanical phenomena do not exist. They are abstractions made either intentionally or from necessity for facilitating our comprehension of things." Though this is not conclusively established, the mechanical theory of nature does seem artificial. There is no reason for believing that an actual mechanism of atoms and molecules as some scientists present, is at the bottom of nature. In fact, recent researches in the electron idea tend to indicate that this is unlikely. But this pictorial mechanical method developed by European thought is a highly serviceable and valuable expedient for generalizing experiences, teaching them, and applying them. It serves a most useful purpose in investigation.

As J. J. Thomson says in his "Corpuscular Theory of Matter,"[2] "From the point of view of the physicist, a theory of matter is a policy rather than a creed; its object is to connect or co-ordinate apparently diverse phenomena, and above all to

[1] "Mechanics," p. 404.
[2] Page 1.

suggest, stimulate and direct experiment." Mechanics will no doubt continue to develop in this way in the future as in the past. As Professor Ziwet says:[1] "It is now pretty generally recognized that Newton's laws of motion including his definition of force are not unalterable laws of thought but merely arbitrary postulates, assumed for the purpose of interpreting natural phenomena in the most simple and adequate manner. . . . It is now coming to be recognized, as researches are made in the electron theory, that the abandonment or generalization of the older mechanics must lead to a more general mechanics. It will probably be non-Newtonian, based on the development of the electron theory including Newton's laws as a special case." Prof. Pearson says:[2] "We must hope to ultimately conceptualize an ether, from the simple structure of which several of the laws of motion postulated for particles of gross matter may directly flow. . . . The customary definitions of mass and force, as well as Newton's statements of the laws of motion, abound in metaphysical obscurities. It is also questionable whether the principles involved in the current statements as to superposition and combination of forces are scientifically correct when applied to atoms and molecules. The hope for future progress lies in clearer conceptions of the nature of ether and of the structure of gross matter."

The history of mechanics in the past indicates that before each step in advance, there is a period of readjustment and of assimilation of previous ideas. We appear to be passing through such a period now. The great generalizations of the law of gravitation and of the principles of conservation of matter and of energy have about done their work of readjustment, and have been assimilated to the previous ideas, forming a body of doctrine as a basis for further progress.

As indications of where this advance may be expected, one may look toward the points at which investigators are dissatisfied with the science, or are not in accord with each other. These are in the direction of the ultimate character of this third concept variously termed matter-energy, ether-squirt,

[1] *Science*, Vol. XXIII, p. 50.
[2] "Grammar of Science," p. 321.

vortex-ring, electron, etc., and in the direction of the law of gravitation. This principle never has fitted in well with the other principles of mechanics. It is found unsatisfactory in that its action seems to be different in kind. It is a convenient generalization, but there is no explanation of how the pull of one body is conducted across space or what conducts it. The principle of action at a distance is not satisfactory. It was not satisfactory to Newton. Progress is to be expected in this direction, and a beginning has already been made with the Electron Theory. It is possible that a more general law may be evolved from the study of potential as expressed in the equation of Poisson.

$$\nabla^2 V = -4\pi\rho.$$

This formula appears to express a mode of conception of natural phenomena which is almost as ultimate as the time and space modes. It may be that a law or mode of conception may be evolved that will include all three modes in one. But though this is foreshadowed in analysis it is not possible yet, to state such a law in words.

As some of the fundamental concepts of formal mechanics, such as matter and the law of gravitation, are not beyond criticism, later advances will very probably reformulate the science. It may indeed be necessary partially to tear down the present system and build it anew on different lines, when the fundamentals are more correctly perceived and comprehended. The science originated with, and developed from a study of gross bodies with motions of considerable amplitude, and the notions thus obtained have been refined and applied in picturing the unseen. The minute operations which produce the large appearances may in the future be pictured as of a different kind and order from the gross things. While the endeavors of the German professors Hertz and Boltzmann in this direction cannot be called successful, they indicate the tendency. We should be careful not to let prejudice in favor of present ideas and methods hamper progress as prejudices have done in the past.

The curious disintegrating effects of radium and uranium and their derivative products, each with a characteristic "rate of decay," tend to weaken the notions of immutability and conservation, and to reinforce the idea that formal mechanics at best gives but a hazy picture of the realities of the world. But it is a model or a picture that can be improved and brought more in accord with a wider and more varied number of phenomena. Though imperfect, its value and utility in applied science and engineering is marvellous. Its economic value is beyond question, and is indeed the reason of its existence, and one of the strong incentives to its improvement.

The "conceptual shorthand," by which the resumé of phenomena is made, will no doubt be improved, but it seems impossible that the fundamental concepts of time and space shall give place. And the third idea which is at present necessary for a formal presentation of the science appears to contain an ultimate element not resolvable into these other two. The future may evolve from electricity or energy a more precise idea of this third fundamental concept and make clearer its connections.

At present, in spite of the fact that some of the generalizations recorded under the head of mechanics are widely applicable in the world of phenomena, we cannot claim that the science comprises a knowledge of the foundations of the world of phenomena, nor indeed a true picture of any reality of the world. The most that may be legitimately claimed is that it gives a tentative mental resumé, as Dr. Mach says, an "aspect" of the world of phenomena which is fairly satisfactory and prodigiously useful and valuable.

Withal we should be on our guard lest our science be too much with us, late and soon, lest we come to reverence these apparent constancies of relation and these serviceable fancies too highly. When in the glories of sunset and rainbow one sees and thinks of nothing but molecules and refractions, then truly there is "little we see in nature that is ours."

The historical review of the development of the Science indicates that it is not essentially in conflict with Philosophy or Religion. It speculates not why, but asks how, and it is only the tyro who finds it incompatible with piety.

While, with some, a mechanical explanation of all nature is an avowed ideal, the scientist who ponders the world of phenomena with an open-mind, cannot but be impressed with a causal activity immanent therein, which is more than blind chance. The universe is more than a fortuitous concourse of molecules. The more we study it, the more need we have to predicate as a cause of the cosmos as a whole, and of its ceaselessly varying infinity of phenomena, an Immanence of Control, incomprehensible to our finite mind.

A BRIEF BIBLIOGRAPHY OF NOTEWORTHY PUBLICATIONS ON THE SCIENCE OF MECHANICS.

Archimedes. Oxford manuscript, edition 1792. German translation of Nizze, 1824.
Arneth, A. Die Geschichte der reinen Mathematik. Stuttgart, 1852.
Arago, F. J. D. Collected works, Paris, 1857, containing an account of various mathematicians of the middle ages and modern times.
Bayma. Molecular Mechanics.
Bernoulli, J. Opera Omnia, Acta Eruditorium, 1693.
Bernoulli, D. Hydrodynamica, 1738.
Baden-Powell, Historical View of the Progress of Physical Science.
Ball, J. R. W. The History of Mathematics.
Bossut, C. Histoire generale des mathematiques, 1810. Cours complet des mathematique, 7 vols., 1801, etc.
Cantor, M. Vorlesungen über die Geschichte der Mathematik. Leipzig.
Cajori, F. A history of Physics.
Clausius. Die mechanische Wärmetheorie, 1876.
Clifford, W. K. Common Sense of the Exact Sciences.
Crookes, Sir Wm. Radiant Matter.
D'Alembert. Traité de Dynamique, 1743.
Delambre, J. B. J. Histoire de l'Astronomie.
Daniel, A. Principles of Physics.
Draper, J. W. Conflict between Religion and Science.
Dühring. Kritische Geschichte der Mechanik.
Duncan, R. K. The New Knowledge.
Euler. Methodus, Opera, 1744 (Leipzig, 1887).
Fleming, Jas. Electronic Theory, *Popular Science Monthly*, 1902.
Fournier. The Electron Theory.
Galileo. Discorsi, 16 vols., Alberi, Florence, 1856.
Gibbs, J. W. Statistical Mechanics.
Gow, J. Short History of Greek Mathematics.
Gunther, S. Vermischte Untersuchungen zur Geschichte der mathematischen Wissenschaften. Leipzig, 1876.
Hamilton. Quaternions.
Hankel, H. Zur Geschichte der Mathematik. Leipzig, 1874.
Heller, A. Geschichte der Physik. Stuttgart, 1882.
Helm. Die Lehre von der Energie.
Hertz. Principien der Mechanik.
Huygens. Horologium Oscillatorium; Opera.
Holman. Matter, Energy, Force and Work.
Jerons. Principles of Science.
Joule. Scientific Papers, 2 vols.
Kaestner, A. G. Geschichte der Mathematik.
Kretschmer. Die physische Erdkunde im Mittelalter.
Kimball. The Physical Properties of Gases.
Lami. Elemens de Mecanique, 1687.
Laplace. Mecanique, Celeste, 1799.

Larmor, Jos. Ether and Matter, 1901.
Lagrange. Mecanique Analytique, 1788.
Lehmann. Molecular Physik (Leipzig, 1889).
Lodge, O. The Ether of Space. Pioneers of Science. On Electrons. The Electrician, 1903.
Love. Theoretical Mechanics, 1897.
MacLaurin. A Complete System of Fluxions.
Marie, M. Histoire des Sciences Math. et Phys. (Paris, 1888).
Mach, E. The Science of Mechanics, 1893.
Mariotte. Traité du Mouvement des Eaux, 1666.
Maspero. The Dawn of Civilization.
Maxwell, C. Matter and Motion, 1892. Theory of Heat, 1897.
Meyer. The Kinetic Theory of Gases (London, 1899).
Michie, P. Elements of Mechanics.
Minchin, G. M. Treatise on Statics.
Mivart, St. G. The Groundwork of Science.
Murchard, F. W. Litteratur der math. Wissenschaften.
Newton, I. Principia.
Nichols, E. F. Physics.
Pascal. Traité, 1662.
Pearson, K. The Grammar of Science.
Planck. Das Princip der Erhaltung der Energie (Leipzig, 1877).
Poggendorff, J. C. Biographisch-Literarisches Handwörterbuch zur Geschichte der exacten Wissenschaften. Leipzig, 1863.
Poisson, S. D. Traité de Mecanique, 1840.
Poinsot. Elemens de Statique, 1877.
Poncelat. Cours de Mecanique.
Poynting & Thomson. Properties of Matter, 1901.
Quetelet, L. A. J. Histoire des Sciences mathematiques et Physiques chez les Belges, 1864. Bruxelles.
Risteen. Molecules and Molecular Theory, 1895.
Rosenthal, G. E. Encyclopædia der Mathematik. Gotha, 1796.
Routh. Rigid Dynamics, 1884. Dynamics of a Particle, 1898.
Rowland, Scientific Papers. Baltimore, 1902.
Stallo, J. B. Modern Physics.
Stevinus, S. Hypomnemata Mathematica, 1608.
Tait, P. C. Recent Advances in Physical Science. Newton's Laws of Motion. Thermodynamics. The Properties of Matter, etc.
Thomson, J. J. The Corpuscular Theory of Matter. The Application of Dynamics to Physics and Chemistry, etc.
Thomson & Tait. Treatise on Natural Philosophy.
Thomson, W., Sir (Kelvin). Lectures and Addresses.
Tylor. Primitive Culture.
Todhunter, I. A History of the Calculus of Variation during Nineteenth Century.
Varignon. Nouvelle Mecanique, 1687.
Wallis. Mechanica Sive de Motu, 1670.
White, A. D. A History of the Warfare of Science and Theology in Christendom, 2 vols.
Wren, C. Lex Natural de Collisione Corporum, 1669.
Whewell. History of the Inductive Sciences.